Solutions Manual

For

Volume One
Classical Mechanics

January 2011

by

David Michael Judd

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Solutions to Problems for Chapter 1

1.) Solution:

a) Recall that

$$1\ mile \equiv 5{,}280\ \ ft \equiv 63{,}360\ inches$$

while

$$1\ inch \equiv 2.54\ \ cm\ \equiv 0.0254\ \ m\ .$$

Substitution gives us

$$1\ mile \equiv 5{,}280\ \ ft \equiv 63{,}360\ \left(0.0254\ \ m\right)\ ,$$

and, therefore,

$$1\ mile \equiv 1{,}609.344\ \ m\ \ ,$$

while

$$1\ \ m \equiv \frac{1}{1{,}609.344}\ mile = 6.213711922 \times 10^{-4}\, mile\ .$$

So, in terms of miles, an astronomical unit is given by

$$1\ \ AU = 1.49597870691 \times 10^{11} \left(6.213711922 \times 10^{-4}\, mile\right)$$
$$\pm 30\ \left(6.213711922 \times 10^{-4}\, mile\right)$$
$$= 9.295580727 \times 10^{7}\ \ miles \pm 0.019\ mile$$
$$= 92{,}955{,}807.27\ miles \pm 0.02\ mile\ \ .$$

b) We can write

$$R_{\oplus} = 6.378135 \times 10^{6} \left(6.213711922 \times 10^{-4}\, mile\right) \pm 5\ \left(6.213711922 \times 10^{-4}\, mile\right)$$
$$= 3.963189 \times 10^{3}\ \ miles \pm .00311\ mile$$
$$= 3{,}963.189\ \ miles \pm .003\ \ mile\ ..$$

c) We can write

$$c = 2.99792458 \times 10^{8} \left(6.213711922 \times 10^{-4}\, mile\right) / \left(1\ \ hour\ /\ 3600\right)$$
$$= 6.70616629 \times 10^{8}\ \ miles\ /\ hour\ .$$

2.) Solution:

a) We can write

$$1\ \ kg \equiv \left(1\ /\ 14.5939\right) slug = 6.852178 \times 10^{-2}\, slug\ \ ,$$

and, therefore,

$$M_{\oplus} = 5.9736 \times 10^{24} \left(6.852178 \times 10^{-2}\, slug\right) = 4.0932 \times 10^{23}\, slugs\ .$$

b) As ratios of like physical quantities are dimensionless, a change in standard unit has no effect on the ratio.

$$\frac{m_p}{m_n} = \frac{1.672621637 \times 10^{-27} \left(6.852178 \times 10^{-2}\, slug\right)}{1.674927211 \times 10^{-27} \left(6.852178 \times 10^{-2}\, slug\right)} = 0.99862\ .$$

c) $m_{e^-} = 9.10938215 \times 10^{-31} \left(6.852178 \times 10^{-2} slug\right) = 6.24191076 \times 10^{-32}\, slug\,.$

3.) Solution:

a) As the speed of light is a constant, we can write

$$\Delta t = \frac{\Delta r}{v} = \frac{2R_{univ}}{c} = \frac{2ct_{univ}}{c} = 2t_{univ} = 2\left(13.7\times10^{9}\ years\right) = 27.4\times10^{9}\ years\,.$$

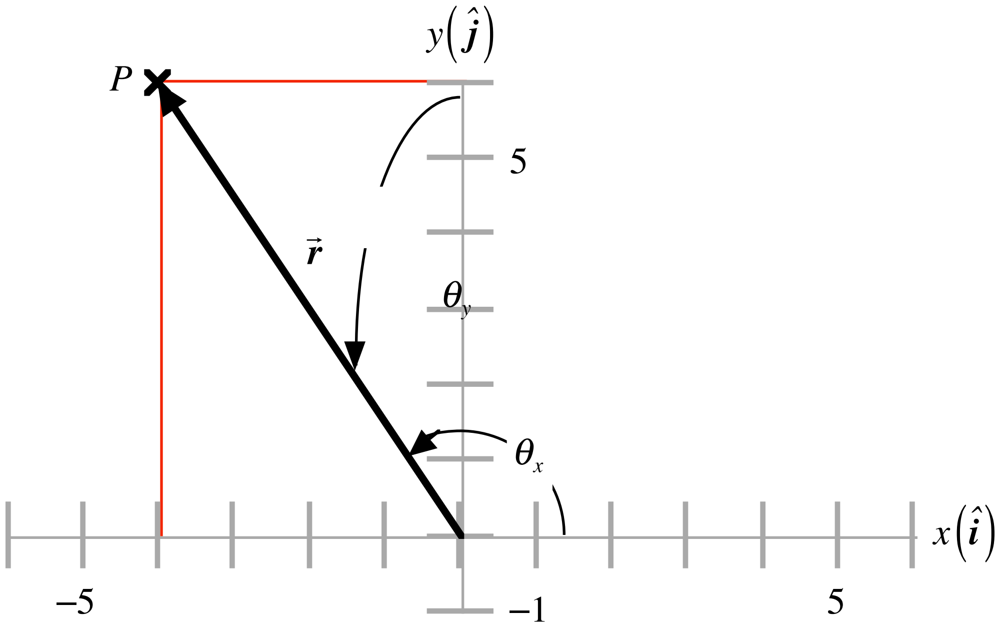
b) $$\Delta t = \frac{\Delta r}{v} = \frac{2R_S}{c} = \frac{2n_s R_\odot}{c} = \frac{2(100)\left(6.955\times10^{8}\,m\right)}{\left(2.99792458\times10^{8}\,m/s\right)} = 464.0\ s \equiv 7.73\ minutes\,.$$

c) $$\Delta t = \frac{\Delta r}{v} = \frac{2R_p}{c} = \frac{2\left(1.2\times10^{-15}\,m\right)}{\left(2.99792458\times10^{8}\,m/s\right)} = 8.0\times10^{-24}\,s\,.$$

Solutions to Problems for Chapter 2

1.) Solution:

a)



b)

$$\vec{\boldsymbol{r}} = r\ \hat{\boldsymbol{r}}\ ,$$

$$r = \sqrt{(-\ 4m)^2 + (6m)^2} = \sqrt{52}\ m = 7.21\ m\ \ .$$

c)

$$\hat{\boldsymbol{r}} = \frac{\vec{\boldsymbol{r}}}{r} = \frac{-\ 4\ m\ \hat{\boldsymbol{i}} + 6\ m\ \hat{\boldsymbol{j}}}{\sqrt{52}\ m} = -\frac{4}{\sqrt{52}}\ \hat{\boldsymbol{i}} + \frac{6}{\sqrt{52}}\ \hat{\boldsymbol{j}} = cos\theta_x\ \hat{\boldsymbol{i}} + cos\,\theta_y\ \hat{\boldsymbol{j}}\ \ .$$

d)

$$\theta_x = cos^{-1}\left[-\ 4/\sqrt{52}\right] = 123.7°\ ,$$

$$\theta_y = cos^{-1}\left[6/\sqrt{52}\right] = 33.7°\ \ .$$

2.) Solution:

b)

$$r = \sqrt{(6m)^2 + (-\ 5m)^2} = \sqrt{61}\ m = 7.81\ m\ \ .$$

c)

$$\hat{\boldsymbol{r}} = \frac{\vec{\boldsymbol{r}}}{r} = \frac{6\ m\ \hat{\boldsymbol{i}} - 5\ m\ \hat{\boldsymbol{j}}}{\sqrt{61}\ m} = \frac{6}{\sqrt{61}}\ \hat{\boldsymbol{i}} - \frac{5}{\sqrt{61}}\ \hat{\boldsymbol{j}} = cos\,\theta_x\ \hat{\boldsymbol{i}} + cos\theta_y\ \hat{\boldsymbol{j}}\ \ .$$

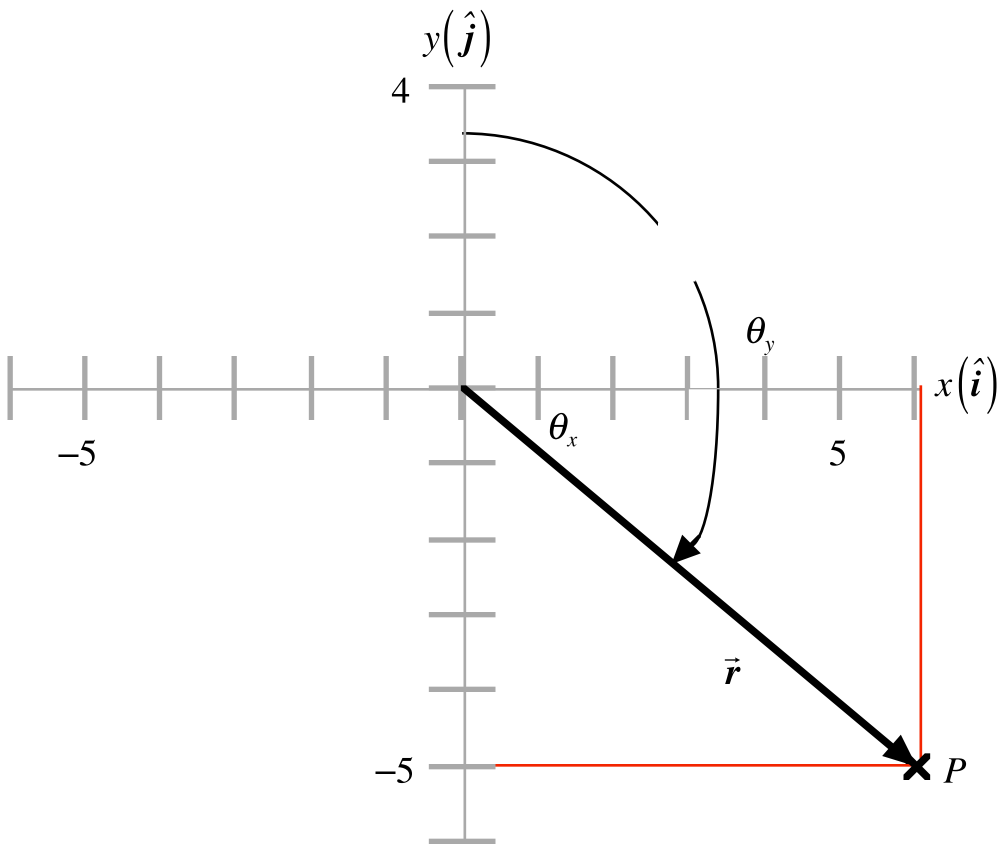
d)

$$\theta_x = cos^{-1}\left[6/\sqrt{61}\right] = 39.8°\ ,$$

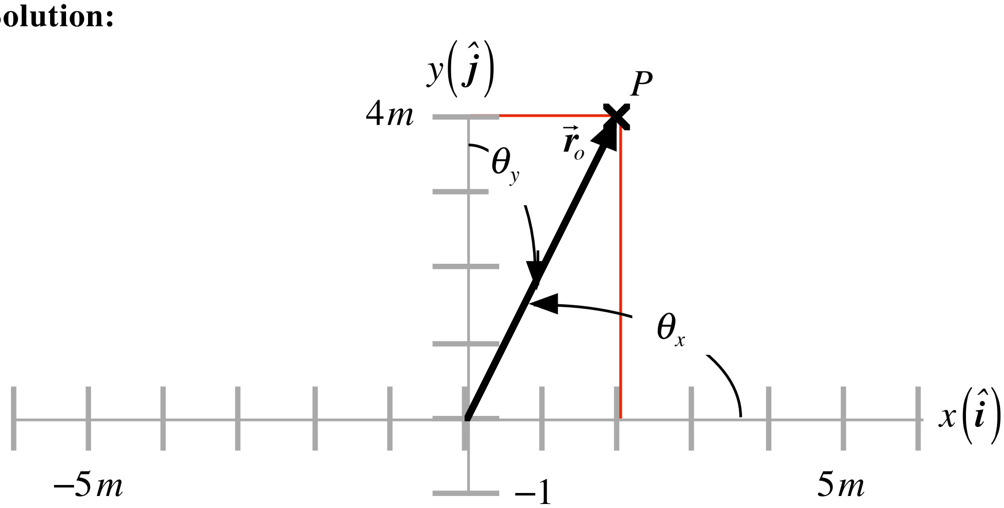
e)

$$\theta_y = cos^{-1}\left[-\ 5/\sqrt{61}\right] = 129.8°\ \ .$$

a)



3.) Solution:



b) $$r_o = \sqrt{(2m)^2 + (4m)^2} = \sqrt{20}\ m = 4.47\ m\ .$$

c) $$\hat{\boldsymbol{r}}_o = \frac{\vec{\boldsymbol{r}}_o}{r_o} = \frac{2\ m\ \hat{\boldsymbol{i}} + 4\ m\ \hat{\boldsymbol{j}}}{\sqrt{20}\ m} = \frac{2}{\sqrt{20}}\ \hat{\boldsymbol{i}} + \frac{4}{\sqrt{20}}\ \hat{\boldsymbol{j}} = cos\theta_x\ \hat{\boldsymbol{i}} + cos\,\theta_y\ \hat{\boldsymbol{j}}\ .$$

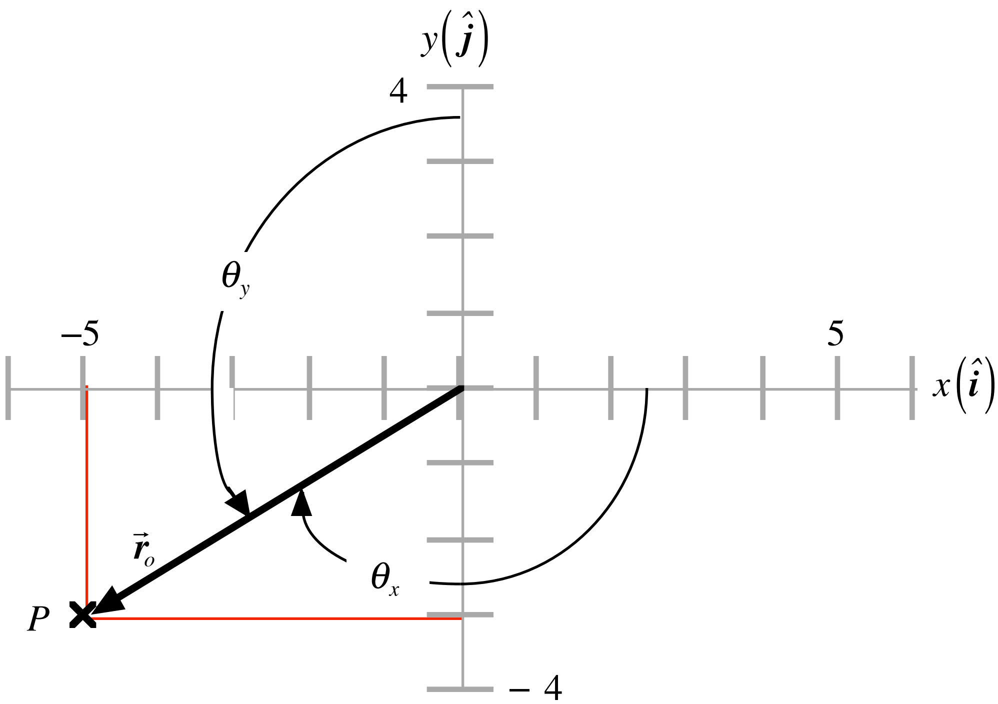
d) $$\theta_x = cos^{-1}\left[2 / \sqrt{20}\right] = 63.4^\circ\ ,$$

e) $\theta_y = cos^{-1}\left[4 / \sqrt{20}\right] = 26.6°$.

a)

4.) Solution:

a)



b) $$r_o = \sqrt{(-5m)^2 + (-3m)^2} = \sqrt{34}\ m = 5.8\ m\ .$$

c) $$\hat{\boldsymbol{r}}_o = \frac{\vec{\boldsymbol{r}}_o}{r_o} = \frac{-5\ m\ \hat{\boldsymbol{i}} - 3\ m\ \hat{\boldsymbol{j}}}{\sqrt{34}\ m} = -\frac{5}{\sqrt{34}}\ \hat{\boldsymbol{i}} - \frac{3}{\sqrt{34}}\ \hat{\boldsymbol{j}} = cos\theta_x\ \hat{\boldsymbol{i}} + cos\theta_y\ \hat{\boldsymbol{j}}\ .$$

d) $$\theta_x = cos^{-1}\left[-5/\sqrt{34}\right] = 149°\ ,$$

e) $$\theta_y = cos^{-1}\left[-3/\sqrt{34}\right] = 121°\ .$$

5.) Solution:

b) $$r = \sqrt{(3m)^2 + (-2m)^2 + (6m)^2} = \sqrt{49}\ m = 7.0\ m\ .$$

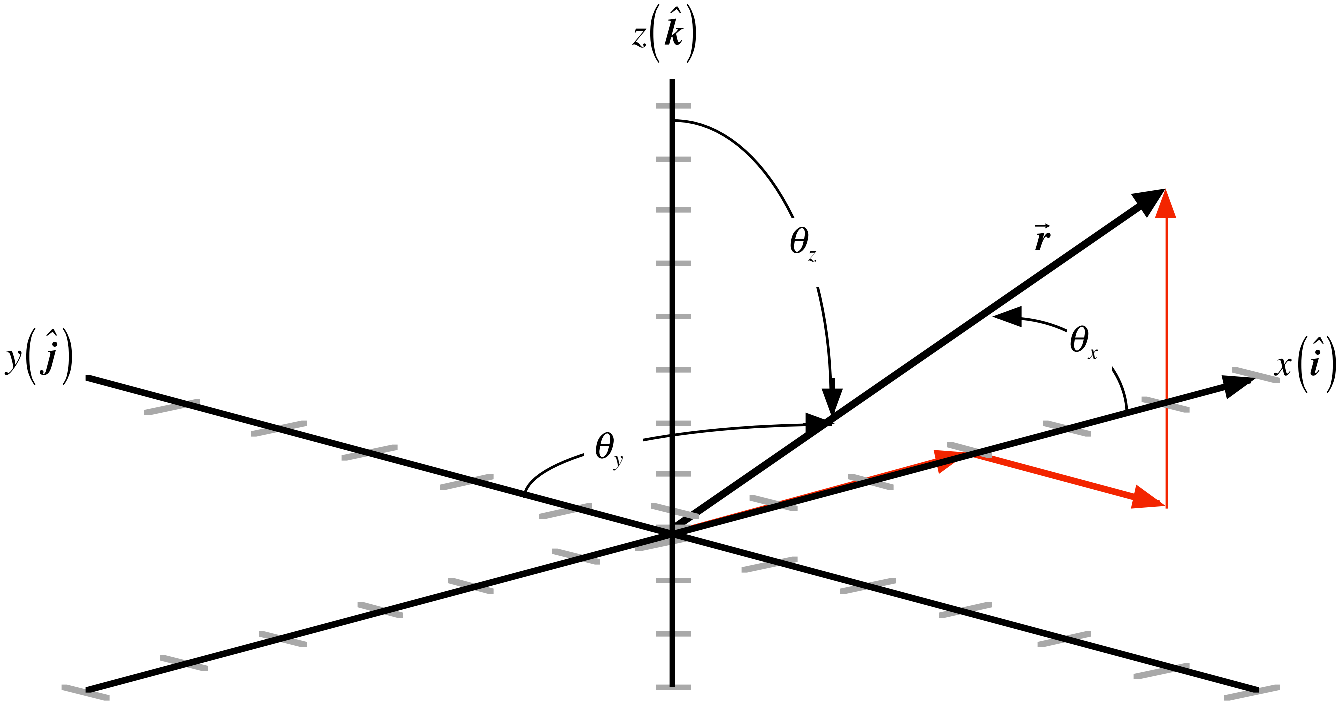
c) $$\hat{\boldsymbol{r}} = \frac{\vec{\boldsymbol{r}}}{r} = \frac{3\ m\ \hat{\boldsymbol{i}} - 2\ m\ \hat{\boldsymbol{j}} + 6\ m\ \hat{\boldsymbol{k}}}{\sqrt{49}\ m} = \frac{3}{\sqrt{49}}\ \hat{\boldsymbol{i}} - \frac{2}{\sqrt{49}}\ \hat{\boldsymbol{j}} + \frac{6}{\sqrt{49}}\ \hat{\boldsymbol{k}}$$
$$= cos\theta_x\ \hat{\boldsymbol{i}} + cos\theta_y\ \hat{\boldsymbol{j}} + cos\theta_z\ \hat{\boldsymbol{k}}\ .$$

d) $$\theta_x = cos^{-1}\left[3/\sqrt{49}\right] = 64.6°\ ,$$

e) $$\theta_y = cos^{-1}\left[-\ 2 / \sqrt{49}\right] = 106.7° \ ,$$

f) $$\theta_z = cos^{-1}\left[6 / \sqrt{49}\right] = 31°$$

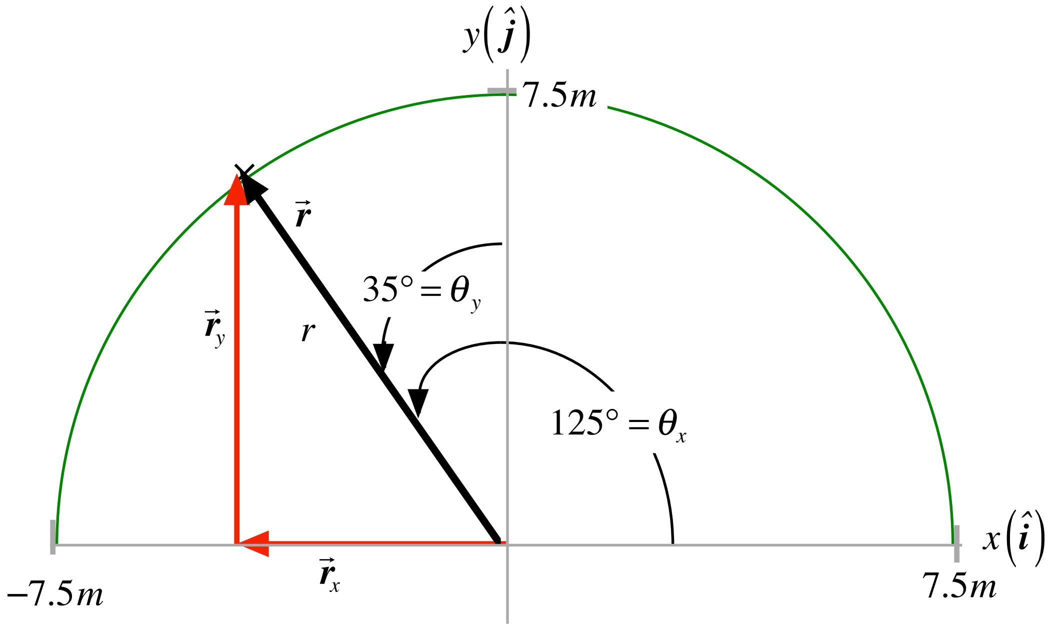
a)



Solutions to Problems for Chapter 3

1.) Solution:

a)



b) We can write

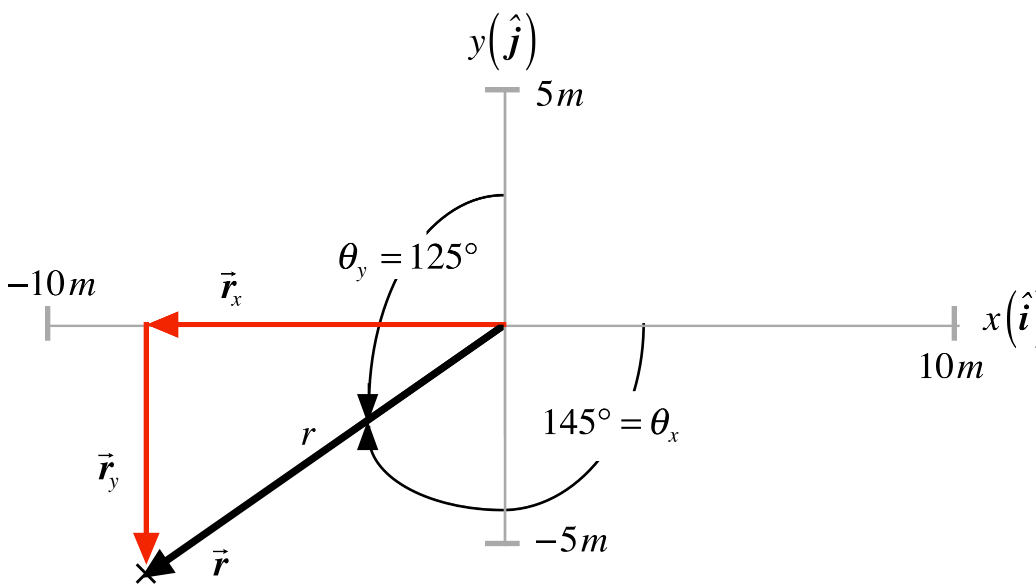
$$\vec{r}_x = r\,cos\theta_x\ \hat{i} = (7.50\ m)(cos125°)\ \hat{i} = -\ 4.30\ m\ \hat{i}\ .$$

c) Also,

$$\vec{r}_y = r\,cos\theta_y\ \hat{j} = (7.50\ m)(cos35°)\ \hat{j} = 6.14\ m\ \hat{j}\ .$$

2.) Solution:

a)



b) We can write

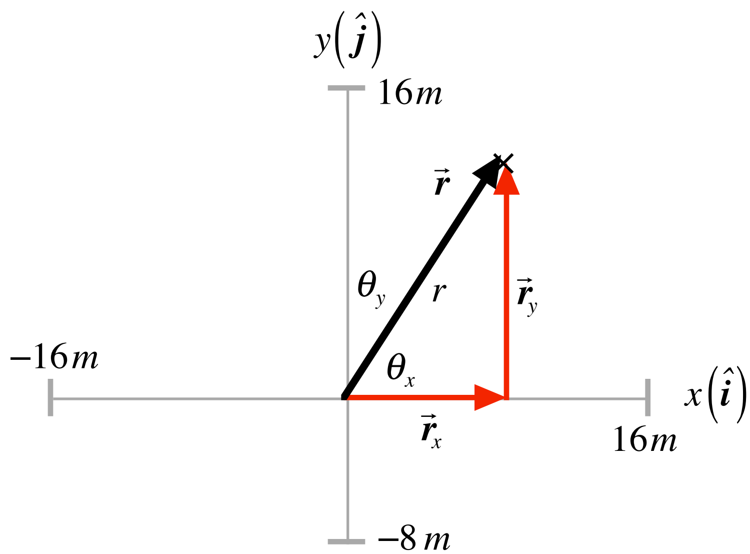
$$\vec{r}_x = r\,cos\theta_x\;\hat{i} = (9.75\;m)(cos145°)\;\hat{i} = -\;7.99\;\;m\;\hat{i}\;.$$

c) Also,

$$\vec{r}_y = r\,cos\theta_y\;\;\hat{j} = (9.75\;m)(cos\,125°)\;\hat{j} = -\;5.59\;\;m\;\;\hat{j}\;\;.$$

3.) Solution:

a)



b) We can write

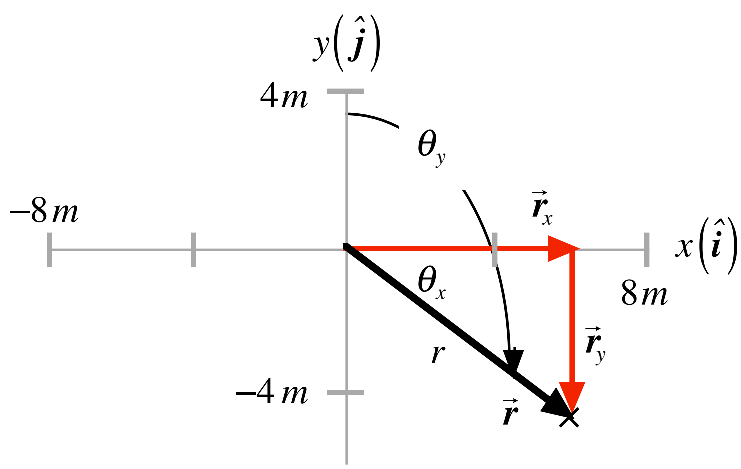
$$\vec{r}_x = r\,cos\theta_x\;\hat{i} = (15.25\;m)(cos57°)\;\hat{i} = 8.31\;m\;\hat{i}\;.$$

c) Also,

$$\vec{r}_y = r\,cos\theta_y\;\;\hat{j} = (15.25\;m)(cos\,33°)\;\hat{j} = 12.79\;\;m\;\;\hat{j}\;\;.$$

4.) Solution:

a)



b) We can write

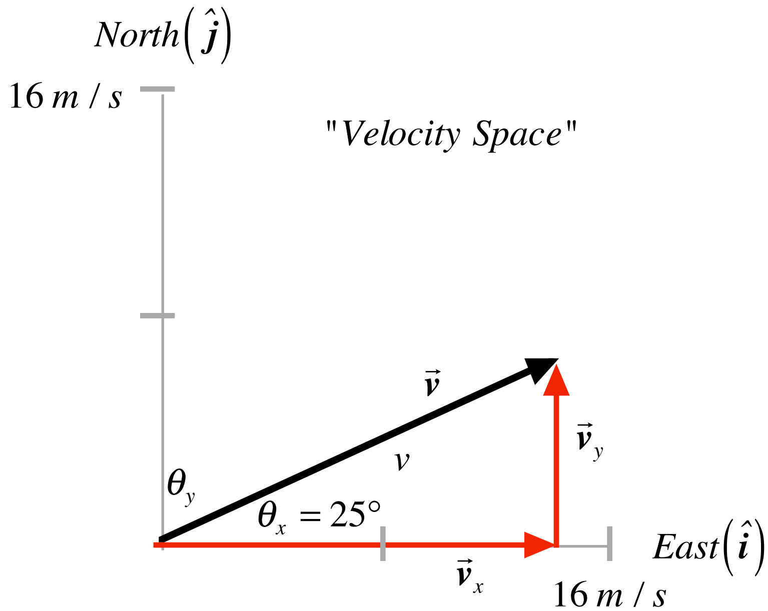
$$\vec{r}_x = r\,cos\theta_x\;\hat{i} = (7.50\;m)(cos\,37°)\;\hat{i} = 5.99\;\;m\;\hat{i}\;.$$

c) Also,

$$\vec{r}_y = r\cos\theta_y \ \hat{j} = (7.50 \ m)(\cos 127°) \ \hat{j} = -\ 4.51 \ m \ \hat{j} \ .$$

5.) **Solution:**

a)



b) We can write

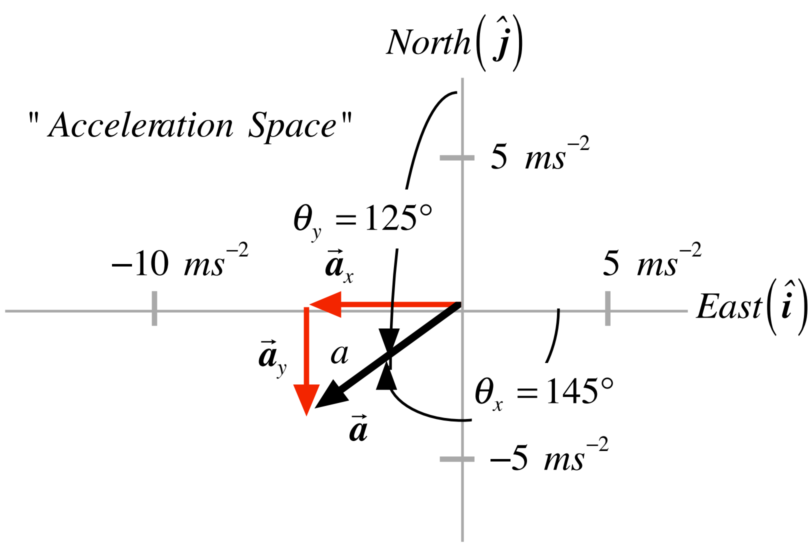
$$\vec{v}_{east} = v\cos\theta_x \ \hat{i} = (15.75 \ m/s)(\cos 25°) \ \hat{i} = 14.27 \ m/s \ \hat{i} \ .$$

c) Also,

$$\vec{v}_{north} = v\cos\theta_y \ \hat{j} = (15.75 \ m/s)(\cos 65°) \ \hat{j} = 6.66 \ m/s \ \hat{j} \ .$$

6.) Solution:

a)



b) We can write

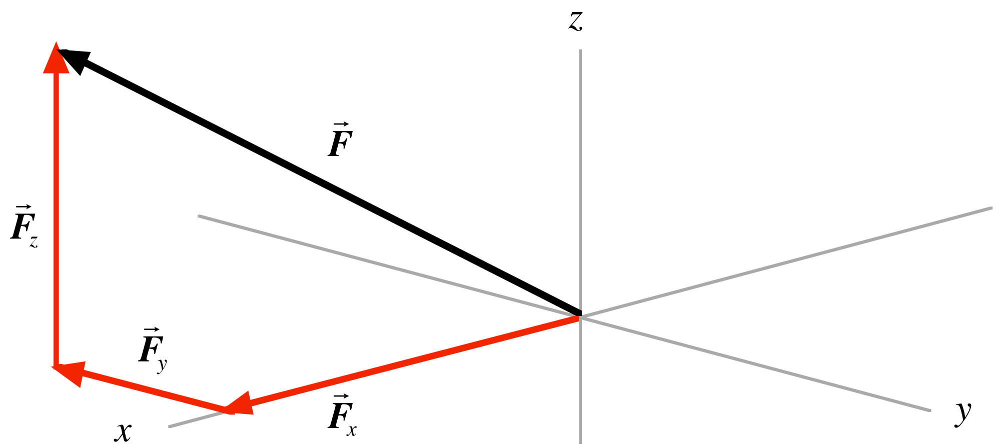
$$\vec{a}_{west} = a\cos\theta_x\ \hat{i} = \left(6.25\ m/s^2\right)\left(\cos 145°\right)\hat{i} = -5.12\ m/s^2\ \hat{i}\ .$$

c) Also,

$$\vec{a}_{south} = a\cos\theta_y\ \hat{j} = \left(6.25\ m/s^2\right)\left(\cos 125°\right)\hat{j} = -3.58\ m/s^2\ \hat{j}\ .$$

7.) Solution:

a)



b) We can write

$$\vec{F}_x = F\cos\theta_x\ \hat{i} = (378\ N)(\cos 40°)\ \hat{i} = 290\ N\ \hat{i}\ .$$

c) $$\vec{F}_y = F\cos\theta_y\ \hat{j} = (378\ N)(\cos 112°)\ \hat{j} = -142\ N\ \hat{j}\ .$$

d) $$\vec{F}_z = F\cos\theta_z\ \hat{k} = (378\ N)(\cos 58.51°)\ \hat{k} = 197\ N\ \hat{k}\ .$$

e) $$\vec{F} = \vec{F}_x + \vec{F}_y + \vec{F}_z = 290\ N\ \hat{i} - 142\ N\ \hat{j} + 197\ N\ \hat{k}\ .$$

Note:

$$(290/142) \approx 2\ \ and\ \ (197/142) \approx 1.4$$

I used this in the construction of the 3-D representation of this force vector.

8.) Solution:

b) We can write

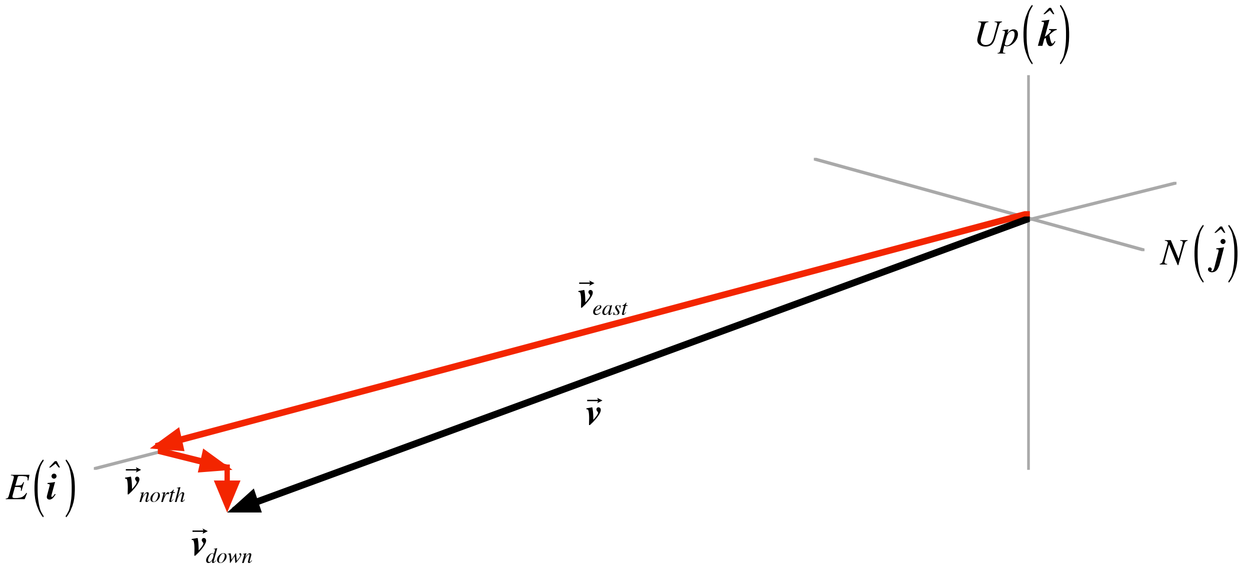
$$\vec{v}_{east} = 112\ m/s\ \hat{i}\ .$$

c) $$\vec{v}_{north} = 6.75\ m/s\ \hat{j}\ .$$

d) $$\vec{v}_{down} = -\ 1.75\ m/s\ \hat{k}\ .$$

e) $$\vec{v} = 112\ m/s\ \hat{i} + 6.75\ m/s\ \hat{j} - 1.75\ m/s\ \hat{k}\ .$$

a)



$Up(\hat{\boldsymbol{k}})$
$N(\hat{\boldsymbol{j}})$
$E(\hat{\boldsymbol{i}})$
$\vec{\boldsymbol{v}}_{east}$
$\vec{\boldsymbol{v}}$
$\vec{\boldsymbol{v}}_{north}$
$\vec{\boldsymbol{v}}_{down}$

Solutions to Problems for Chapter 4

1.) Solution:

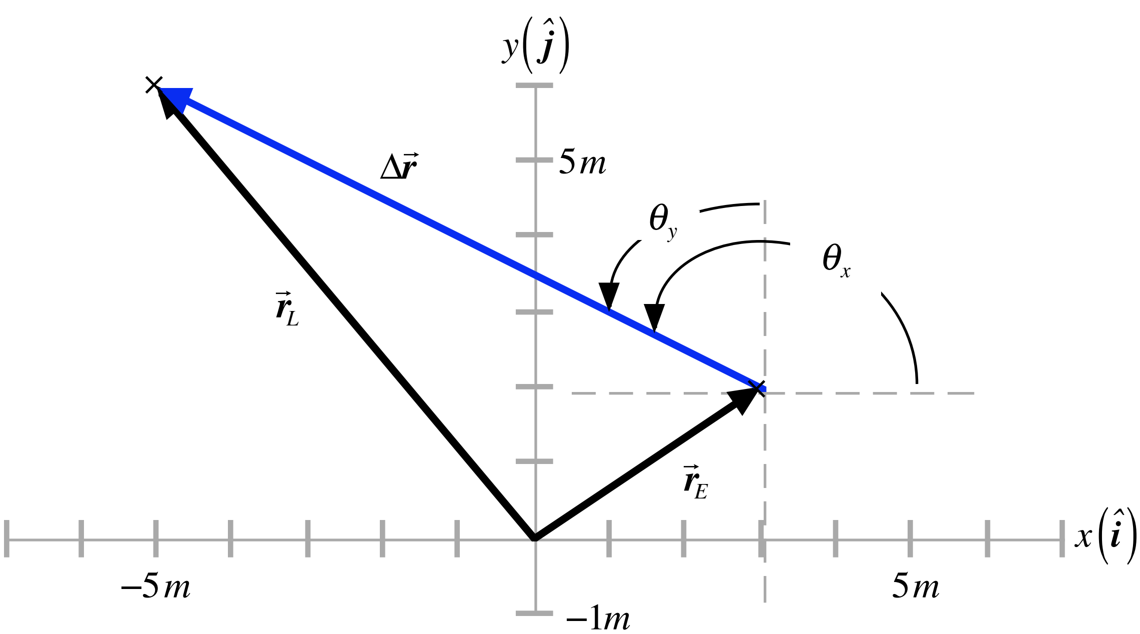
b) We can write

$$\Delta\vec{r} = \Delta r\ \hat{\Delta r} = \vec{r}_L - \vec{r}_E$$
$$= \left[-5.000\ m\ \hat{i} + 6.000\ m\ \hat{j}\right] - \left[3.000\ m\ \hat{i} + 2.000\ m\ \hat{j}\right]$$
$$= -8.000\ m\ \hat{i} + 4.000\ m\ \hat{j}\ .$$
$$\Delta r = \sqrt{(-8\ m)^2 + (4\ m)^2} = \sqrt{80}\ m = 4\sqrt{5}\ m = 8.944\ m\ .$$

c) $$\hat{\Delta r} = \frac{\Delta\vec{r}}{\Delta r} = -\frac{8}{\sqrt{80}}\ \hat{i} + \frac{4}{\sqrt{80}}\ \hat{j} = cos\,\theta_x\ \hat{i} + cos\,\theta_y\ \hat{j}\ .$$

d) $$\theta_x = cos^{-1}\left[-8/\sqrt{80}\right] = 153.4°\ .$$
$$\theta_y = cos^{-1}\left[4/\sqrt{80}\right] = 63.4°\ .$$

a)



2.) Solution:

b) $$\Delta\vec{r} = \Delta r\ \hat{\Delta r} = \vec{r}_2 - \vec{r}_1 = \left[2.00\,m\ \hat{i} + 4.00\,m\ \hat{j}\right] - \left[5.00\,m\ \hat{i} - 6.00\,m\ \hat{j}\right]$$
$$= -3.00\,m\ \hat{i} + 10.00\,m\ \hat{j}\ .$$

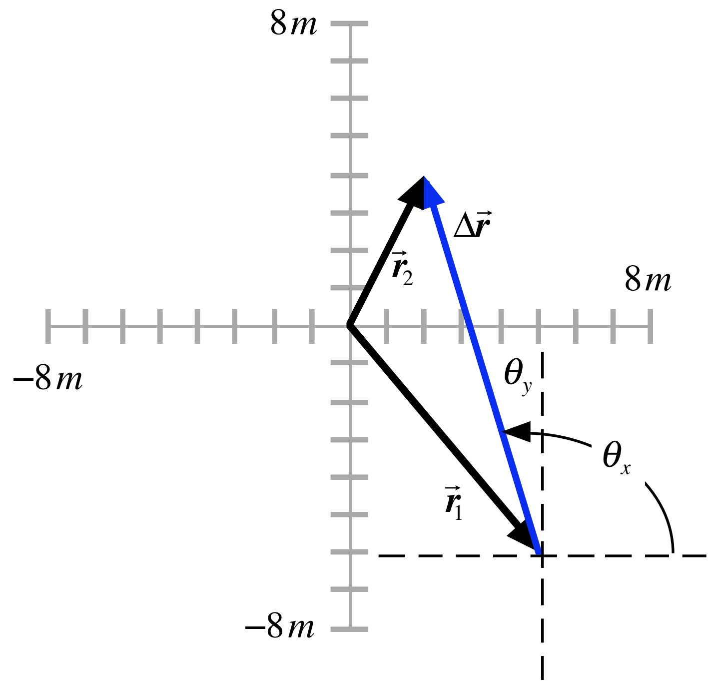
c) $$\Delta r = \sqrt{(-3\ m)^2 + (10\ m)^2} = \sqrt{109}\ m = 10.44\ m\ .$$

d) $$\hat{\Delta r} = \frac{\Delta\vec{r}}{\Delta r} = -\frac{3}{\sqrt{109}}\,\hat{i} + \frac{10}{\sqrt{109}}\,\hat{j} = \cos\theta_x\,\hat{i} + \cos\theta_y\,\hat{j}\ .$$

e) $$\theta_x = \cos^{-1}\left[-3/\sqrt{109}\right] = 106.7°\ .$$

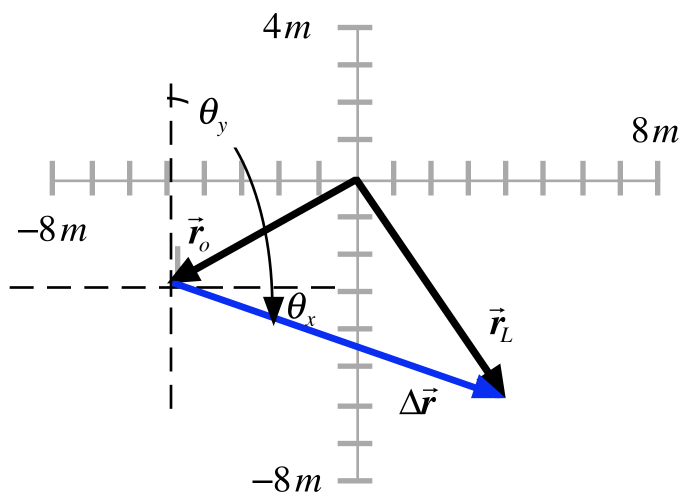
f) $$\theta_y = \cos^{-1}\left[10/\sqrt{109}\right] = 16.7°\ .$$

a)



3.) Solution:

a)



b) $$\Delta\vec{r} = \Delta r\ \hat{\Delta r} = \vec{r}_L - \vec{r}_o = \left[\,4.00\,m\ \hat{i} - 6.00\,m\ \hat{j}\,\right] - \left[-5.00\,m\ \hat{i} - 3.00\,m\ \hat{j}\,\right]$$
$$= 9.00\,m\ \hat{i} - 3.00\,m\ \hat{j}\ .$$

c) $$\Delta r=\sqrt{(9\ m)^2+(-3\ m)^2}=\sqrt{90}\ m=9.49\ m\ .$$

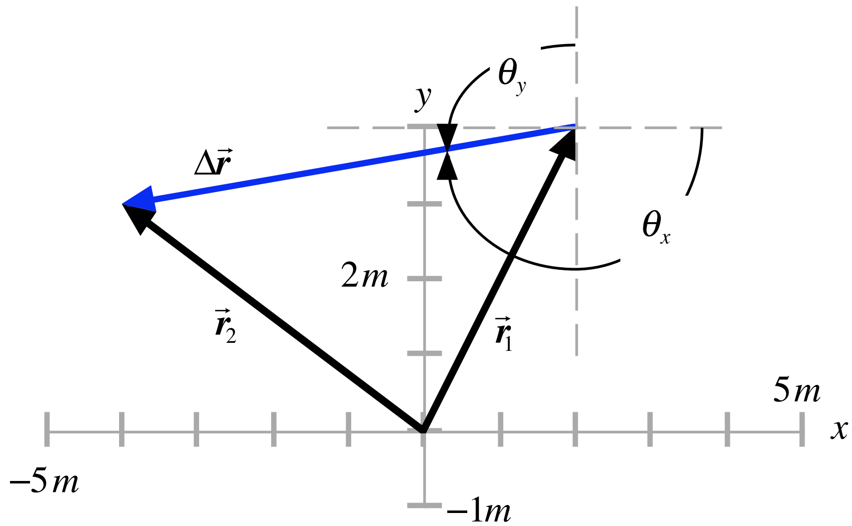
d) $$\hat{\Delta r}=\frac{\Delta\vec{r}}{\Delta r}=\frac{9}{\sqrt{90}}\,\hat{\boldsymbol{i}}-\frac{3}{\sqrt{90}}\,\hat{\boldsymbol{j}}=cos\theta_x\ \hat{\boldsymbol{i}}+cos\theta_y\ \hat{\boldsymbol{j}}\ .$$

e) $$\theta_x=cos^{-1}\left[9/\sqrt{90}\right]=18.4°\ .$$

f) $$\theta_y=cos^{-1}\left[-3/\sqrt{90}\right]=108.4°.$$

4.) Solution:

a)



b) $$\Delta\vec{r}=\Delta r\ \hat{\Delta r}=\vec{r}_2-\vec{r}_1=\left[-4\,m\ \hat{\boldsymbol{i}}+3\,m\,\hat{\boldsymbol{j}}\right]-\left[2\,m\ \hat{\boldsymbol{i}}+4\,m\ \hat{\boldsymbol{j}}\right]$$
$$=-6\ m\ \hat{\boldsymbol{i}}-1m\ \hat{\boldsymbol{j}}\ .$$

c) $$\Delta r=\sqrt{(-6\ m)^2+(-1\ m)^2}=\sqrt{37}\ m=6.1\ m\ .$$

d) $$\hat{\Delta r}=\frac{\Delta\vec{r}}{\Delta r}=-\frac{6}{\sqrt{37}}\,\hat{\boldsymbol{i}}-\frac{1}{\sqrt{37}}\,\hat{\boldsymbol{j}}=cos\theta_x\ \hat{\boldsymbol{i}}+cos\,\theta_y\ \hat{\boldsymbol{j}}\ .$$

e) $$\theta_x=cos^{-1}\left[-6/\sqrt{37}\right]=170.5°\ .$$

f) $$\theta_y=cos^{-1}\left[-1/\sqrt{37}\right]=99.5°.$$

5.) Solution:

a) $$\vec{r}=r\ \hat{\boldsymbol{r}}=\left[-6\ m\ \hat{\boldsymbol{i}}+2\ m\ \hat{\boldsymbol{j}}\right]-\left[4\ m\ \hat{\boldsymbol{i}}+3\ m\ \hat{\boldsymbol{j}}\right]=-10\ m\ \hat{\boldsymbol{i}}-1\ m\ \hat{\boldsymbol{j}}\ .$$
$$r=\sqrt{(-10\ m)^2+(-1\ m)^2}=\sqrt{101}\ m=10.05\ m\ .$$
$$\hat{\boldsymbol{r}}=\vec{r}/r=-\left(10/\sqrt{101}\right)\hat{\boldsymbol{i}}-\left(1/\sqrt{101}\right)\hat{\boldsymbol{j}}=cos\theta_x\ \hat{\boldsymbol{i}}+cos\,\theta_y\ \hat{\boldsymbol{j}}\ .$$

$$\theta_x = cos^{-1}\left[-10/\sqrt{101}\right] = 174.3^\circ .$$

$$\theta_y = cos^{-1}\left[-1/\sqrt{101}\right] = 95.7^\circ .$$

b) $$\vec{\boldsymbol{r}} = r\ \hat{\boldsymbol{r}} = \left[2\ m\ \hat{\boldsymbol{j}} - 4\ m\ \hat{\boldsymbol{k}}\right] - \left[3\ m\ \hat{\boldsymbol{j}} + 5\ m\ \hat{\boldsymbol{k}}\right] = -1\ m\ \hat{\boldsymbol{j}} - 9\ m\ \hat{\boldsymbol{k}}\ .$$

$$r = \sqrt{(-1\ m)^2 + (-9\ m)^2} = \sqrt{82}\ m = 9.06\ m\ .$$

$$\hat{\boldsymbol{r}} = \frac{\vec{\boldsymbol{r}}}{r} = -\frac{1}{\sqrt{82}}\ \hat{\boldsymbol{j}} - \frac{9}{\sqrt{82}}\ \hat{\boldsymbol{k}} = cos\theta_y\ \hat{\boldsymbol{j}} + cos\theta_z\ \hat{\boldsymbol{k}}\ .$$

$$\theta_y = cos^{-1}\left[-1/\sqrt{82}\right] = 96.3^\circ .$$

$$\theta_z = cos^{-1}\left[-9/\sqrt{82}\right] = 173.7^\circ .$$

c) $$\vec{\boldsymbol{r}} = r\ \hat{\boldsymbol{r}} = \left[5\ m\ \hat{\boldsymbol{i}} + 5\ m\ \hat{\boldsymbol{j}} - 4\ m\ \hat{\boldsymbol{k}}\right] - \left[-3\ m\ \hat{\boldsymbol{i}} + 4\ m\ \hat{\boldsymbol{j}} + 2\ m\ \hat{\boldsymbol{k}}\right]$$

$$= 8\ m\ \hat{\boldsymbol{i}} + 1\ m\ \hat{\boldsymbol{j}} - 6\ m\ \hat{\boldsymbol{k}}\ .$$

$$r = \sqrt{(8\ m)^2 + (1\ m)^2 + (-6\ m)^2} = \sqrt{101}\ m = 10.05\ m\ .$$

$$\hat{\boldsymbol{r}} = \frac{\vec{\boldsymbol{r}}}{r} = \frac{8}{\sqrt{101}}\ \hat{\boldsymbol{i}} + \frac{1}{\sqrt{101}}\ \hat{\boldsymbol{j}} - \frac{6}{\sqrt{101}}\ \hat{\boldsymbol{k}} = cos\,\theta_x\ \hat{\boldsymbol{i}} + cos\,\theta_y\ \hat{\boldsymbol{j}} + cos\,\theta_z\ \hat{\boldsymbol{k}}\ .$$

$$\theta_x = cos^{-1}\left[8/\sqrt{101}\right] = 37.2^\circ .$$

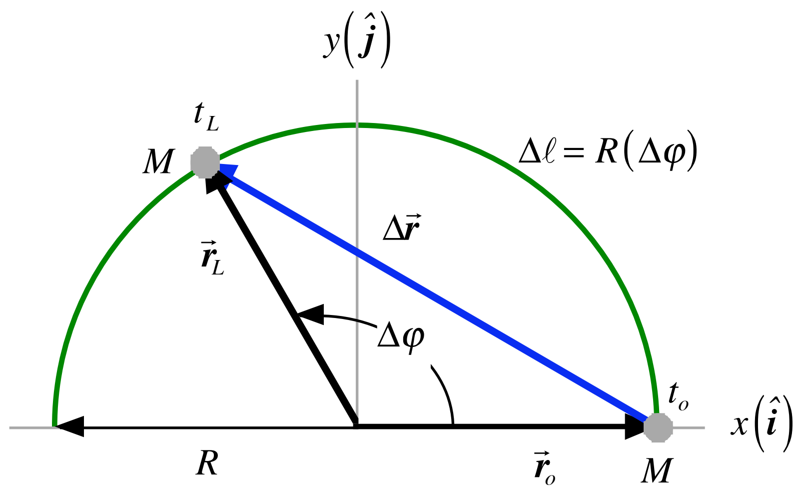
$$\theta_y = cos^{-1}\left[1/\sqrt{101}\right] = 84.3^\circ .$$

$$\theta_z = cos^{-1}\left[-6/\sqrt{101}\right] = 126.7^\circ .$$

Solutions to Problems for Chapter 5

1.) Solution:

a)



b) $$\Delta\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_L - \vec{\boldsymbol{r}}_o = \left[-1.5000\ m\ \hat{\boldsymbol{i}} + 2.5981\ m\ \ \hat{\boldsymbol{j}}\right] - \left[3.0000\ m\ \hat{\boldsymbol{i}}\right]$$
$$= -4.5000\ m\ \hat{\boldsymbol{i}} + 2.5981\ m\ \ \hat{\boldsymbol{j}}\ .$$

c) $$\vec{\boldsymbol{v}}_{ave} = \frac{\Delta\vec{\boldsymbol{r}}}{\Delta t} = \frac{-4.5000\ m\ \hat{\boldsymbol{i}} + 2.5981\ m\ \ \hat{\boldsymbol{j}}}{1\ s} = -4.5000\ \frac{m}{s}\ \hat{\boldsymbol{i}} + 2.5981\ \frac{m}{s}\ \ \hat{\boldsymbol{j}}\ .$$

d) $$v_{ave} = \sqrt{(-4.5000\ m/s)^2 + (2.5981\ m/s)^2} = 5.1962\ m/s\ .$$

e) $$\Delta\ell = R(\Delta\varphi) = (3.000\ m)\left[120(\pi/180)\right] = 6.2832\ m\ .$$

f) $$\mathcal{S}_{ave} = \frac{\Delta\ell}{\Delta t} = \frac{6.2832\ m}{1\ s} = 6.2832\ \frac{m}{s}\ .$$

g) $$\mathcal{S}_{ave} > v_{ave}\ .$$

The magnitude of the average velocity represents the constant speed of an object that moves from the initial position to the later position via a straight line path joining these points. That path length, in this case, is shorter than the length of the circular arc the object actually took.

2.) Solution:

b) $$\Delta\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_2 - \vec{\boldsymbol{r}}_1 = \left[R\cos 135°\ \hat{\boldsymbol{i}} + R\sin 135°\ \ \hat{\boldsymbol{j}}\right] - \left[R\ \hat{\boldsymbol{i}}\right]$$
$$= R(\cos 135° - 1)\ \hat{\boldsymbol{i}} + R\sin 135°\ \ \hat{\boldsymbol{j}} = -4.951\ m\ \hat{\boldsymbol{i}} + 2.051\ m\ \hat{\boldsymbol{j}}\ .$$

c) $$\vec{\boldsymbol{v}}_{ave} = \frac{\Delta\vec{\boldsymbol{r}}}{\Delta t} = \frac{-4.951\ m\ \hat{\boldsymbol{i}} + 2.051\ m\ \hat{\boldsymbol{j}}}{0.500\ s} = -9.902\ \frac{m}{s}\ \hat{\boldsymbol{i}} + 4.102\ \frac{m}{s}\ \ \hat{\boldsymbol{j}}\ .$$

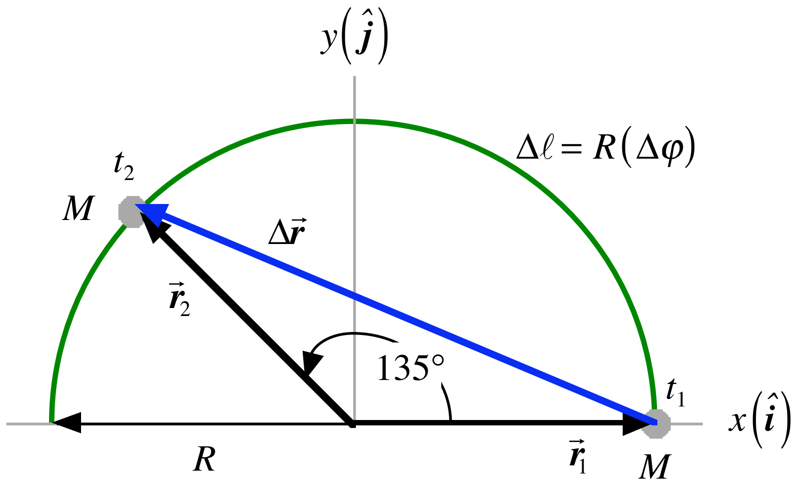
d) $$v_{ave} = \sqrt{(-9.902\ m/s)^2 + (4.102\ m/s)^2} = 10.718\ m/s\ .$$

e) $$\Delta\ell = R(\Delta\varphi) = (2.900\ m)[135(\pi/180)] = 6.833\ m.$$

f) $$s_{ave} = \frac{\Delta\ell}{\Delta t} = \frac{6.833\ m}{0.5\ s} = 13.666\ \frac{m}{s}.$$

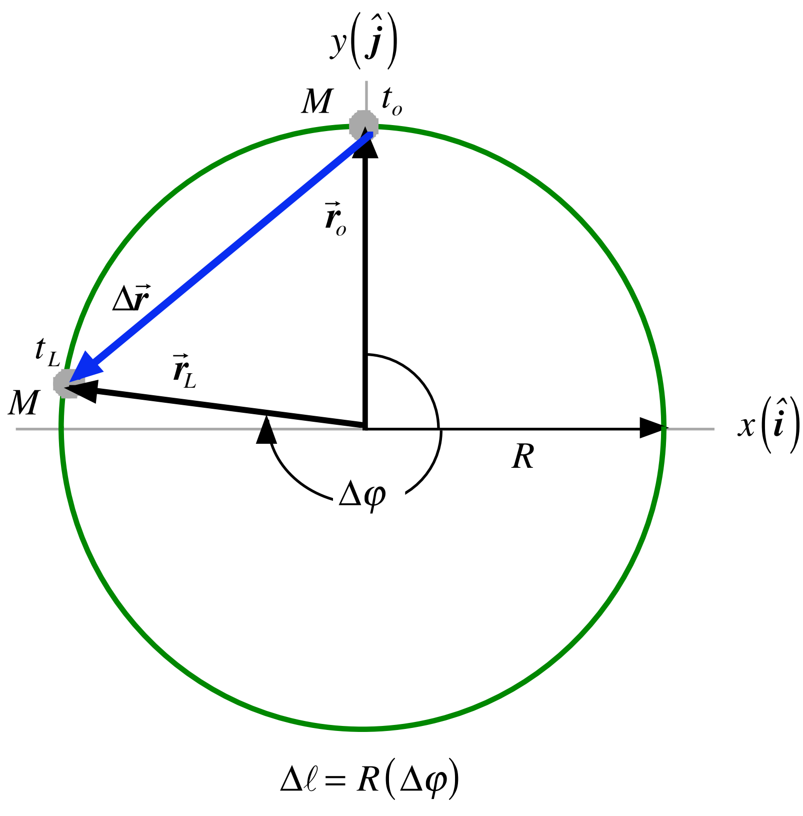
g) $$s_{ave} > v_{ave}.$$

a)



3.) Solution:

a)



b) First, note

$$\frac{\Delta\varphi}{360°}=\frac{2.50\ s}{3.25\ s}\ \rightarrow\ \Delta\varphi=\left[\frac{2.50\ s}{3.25\ s}\right]360°=276.9231°\ .$$

$$\Delta\vec{r}=\vec{r}_L-\vec{r}_o=\left[-R\,cos6.9231°\ \hat{i}+R\,sin6.9231°\ \ \hat{j}\right]-\left[R\ \hat{j}\right]$$

$$=-R\,cos6.9231°\ \hat{i}-R\left(1-sin6.9231°\right)\ \hat{j}=-\ 3.475\ \ m\ \hat{i}-3.078\ \ m\ \ \hat{j}\ .$$

$$\vec{v}_{ave}=\frac{\Delta\vec{r}}{\Delta t}=\frac{-\ 3.474\ \ m\ \hat{i}-3.078\ \ m\ \ \hat{j}}{2.500\ \ s}=-\ 1.390\ \frac{m}{s}\ \hat{i}-1.231\ \frac{m}{s}\ \ \hat{j}\ .$$

c) $$v_{ave}=\sqrt{\left(-\ 1.390\ \ m/s\right)^2+\left(-1.231\ \ m/s\right)^2}=1.857\ \ m/s\ .$$

d) $$\hat{v}=\frac{\vec{v}}{v}=-\frac{1.390}{1.857}\ \hat{i}-\frac{1.231}{1.857}\ \hat{j}=-\ 0.7485\ \hat{i}-0.6629\ \hat{j}=cos\,\theta_x\ \hat{i}+sin\,\theta_y\ \hat{j}\ .$$

e) $$\Delta\ell=R\left(\Delta\varphi\right)=\left(3.50\ \ m\right)\left[\left(360\left(2.5/3.25\right)\right)\left(\pi/180\right)\right]=16.916\ \ m\ .$$

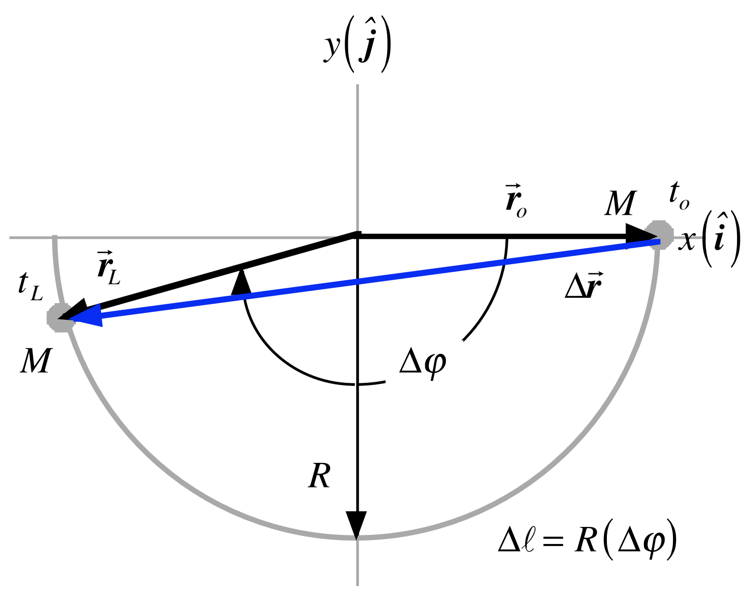
f) $$s_{ave}=\frac{\Delta\ell}{\Delta t}=\frac{16.916\ \ m}{2.5\ \ s}=6.767\ \frac{m}{s}\ .$$

g) $$s_{ave}>v_{ave}\ .$$

In this scenario, the magnitude of the average velocity is a very bad description of the motion.

4.) Solution:

a)



f) First, note

$$\frac{\tau}{2}=\frac{\pi R}{v}=\frac{\pi\left(2{,}400\ \ m\right)}{\left(90.77\ \ m/s\right)}=83.065\ \ s\ ,$$

and, therefore, $\Delta t<\tau/2$, and

$$\Delta\varphi = 180° - tan^{-1}\left|\frac{0.25897}{0.96589}\right| = 165° \equiv 2.8796\ rad\ .$$

b) $$\Delta\vec{r} = \vec{r}_L - \vec{r}_o = \left[-0.96593R\ \hat{i} - 0.25882\,R\ \hat{j}\right] - \left[R\ \hat{i}\right]$$
$$= -1.96593R\ \hat{i} - 0.25882\,R\ \hat{j} = -\ 4.718\times10^3 m\ \hat{i} - 6.212\times10^2\ \ m\ \hat{j}\ .$$

c)
$$\vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t} = \left[\frac{-\ 4.718\times10^3\ \ \hat{i} - 6.212\times10^2\ \ \hat{j}}{76.139}\right]\frac{m}{s} = -\ 61.97\ \frac{m}{s}\ \hat{i} - 8.16\ \frac{m}{s}\ \ \hat{j}.$$

d) $$v_{ave} = \sqrt{\left(-\ 61.97\ \ m/s\right)^2 + \left(-\ 8.16\ m/s\right)^2} = 62.51\ \ m/s\ .$$

e) $$\hat{v} = \frac{\vec{v}}{v} = \vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t} = -\frac{61.97}{62.51}\ \hat{i} - \frac{8.16}{62.51}\ \ \hat{j} = -\ 0.9914\ \hat{i} - 0.1305\ \ \hat{j}\ .$$

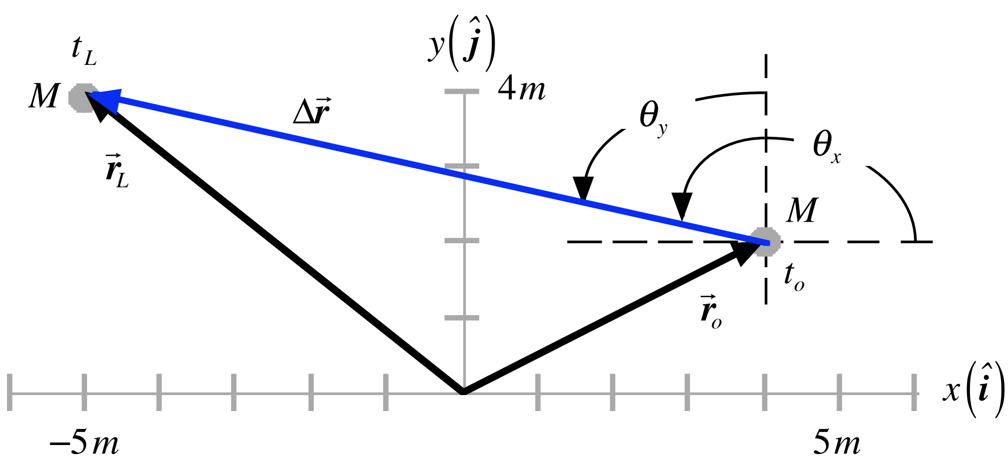
g) $$\Delta\ell = R\left(\Delta\varphi\right) = \left(2400\ \ m\right)\left[2.8796\ rad\right] = 6.911\times10^3\ \ m\ .$$

g) $$s_{ave} = \frac{\Delta\ell}{\Delta t} = \frac{6.911\times10^3\ \ m}{76.139\ \ s} = 90.77\ \ \frac{m}{s}\ \ .$$

h) $$\frac{s_{ave}}{v_{ave}} = \frac{90.77\ \ m/s}{62.51\ m/s} = 1.452\ \ .$$

5.) Solution:

a)



b) $$\Delta\vec{r} = \vec{r}_L - \vec{r}_o = \left[-5\,m\ \hat{i} + 4\ m\ \hat{j}\right] - \left[4\ m\ \hat{i} + 2\,m\ \hat{j}\right] = -\ 9\,m\ \hat{i} + 2\,m\ \hat{j}\ \ .$$

c) $$\Delta r = \sqrt{\left(-\ 9m\right)^2 + \left(2\ \ m\right)^2} = \sqrt{85}\ \ m = 9.2\ \ m\ \ .$$

d) $$\vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t} = \frac{-\ 9\,m\ \hat{i} + 2\,m\ \hat{j}}{4\ \ s} = -\ 2.25\ \ \frac{m}{s}\ \hat{i} + 0.50\ \ \frac{m}{s}\ \ \ \hat{j}\ .$$

e) $$v_{ave} = \sqrt{\left(-\ 2.25\ \ m/s\right)^2 + \left(0.50\ \ m/s\right)^2} = 2.305\ \ m/s\ .$$

$$\hat{\boldsymbol{v}}_{ave} = \frac{\vec{\boldsymbol{v}}_{ave}}{v_{ave}} = \frac{-\ 2.25\ m/s\ \hat{\boldsymbol{i}} + 0.50\ m/s\ \ \hat{\boldsymbol{j}}}{2.305\ m/s} = -\ 0.9761\ \hat{\boldsymbol{i}} + 0.2169\ \ \hat{\boldsymbol{j}}\ .$$

$$\theta_x = cos^{-1}\left[-\ 0.9761\right] = 167.5°.$$

$$\theta_y = cos^{-1}\left[0.2169\right] = 77.5°\ .$$

Solutions to Problems for Chapter 6

1.) Solution:

a) $$p_T = M_T v = \left(1\times 10^4 kg\right)\left(20\ m/s\right) = 2\times 10^5\ kg\ m/s\,.$$

b) $$K_T = \frac{1}{2}M_T v^2 = \frac{p_T^2}{2M_T} = \frac{\left(2\times 10^5 kg\ m/s\right)^2}{2\left(1\times 10^4\ kg\right)} = 2\times 10^6\ J\ .$$

c) i) $$p_T = M_T v = p_v = m v_v\ ,$$

$$v_v = \left[\frac{M_T}{m}\right]v = \left[\frac{M_T}{M_T/10}\right]v = 10v = 10\left(20\ m/s\right) = 200\ m/s\ .$$

ii) $$K_T = \frac{1}{2}M_T v^2 = K_v = \frac{1}{2}m v_v^2\ ,$$

$$v_v = \left[\frac{M_T}{m}\right]^{1/2} v = \left[\frac{M_T}{M_T/10}\right]^{1/2} v = \left[10\right]^{1/2} v = \left[10\right]^{1/2}\left(20\ \frac{m}{s}\right) = 63.25\ \frac{m}{s}.$$

2.) Solution:

a) $$p_b = M_b v = \left(0.05\ kg\right)\left(400\ m/s\right) = 20\ kg\ m/s\ .$$

b) $$K_b = \left(1/2\right)M_b v^2 = \left(0.5\right)\left(0.05\ kg\right)\left(400\ m/s\right)^2 = 4{,}000\ J\ .$$

3.) Solution:

a) $$p = Mv = \left(4.4\ kg\right)\left(1{,}400\ m/s\right) = 6{,}160\ kg\ m/s\ .$$

b) $$K = \left(1/2\right)Mv^2 = \left(0.5\right)\left(4.4\ kg\right)\left(1{,}400\ m/s\right)^2 = 4.312\times 10^6\ J\ .$$

4.) Solution:

a) $$p = Mv = \left(4.4\times 10^5\ kg\right)\left(81.0\ m/s\right) = 3.56\times 10^7\ kg\ m/s\ .$$

b) $$K = \left(1/2\right)Mv^2 = \left(0.5\right)\left(4.4\times 10^5\ kg\right)\left(81.0\ m/s\right)^2 = 1.44\times 10^9\ J\ .$$

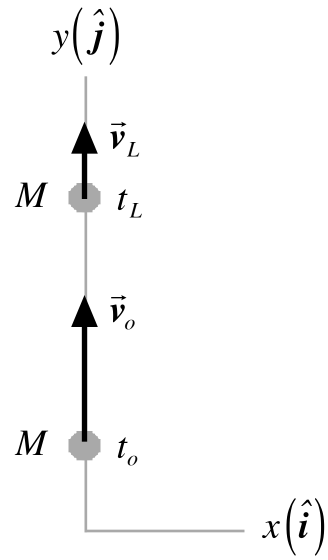
5.) Solution:

a) $$p_{\oplus,orb} = M_\oplus v = \left(5.98\times 10^{24} kg\right)\left(2.98\times 10^4\ m/s\right) = 1.78\times 10^{29}\ kg\ m/s\ .$$

b) $$K_\oplus = \left(1/2\right)M_\oplus v^2 = \left(0.5\right)\left(5.97\times 10^{24} kg\right)\left(2.98\times 10^4\ m/s\right)^2 = 2.65\times 10^{33}\ J.$$

Solutions to Problems for Chapter 7

1.) Solution:



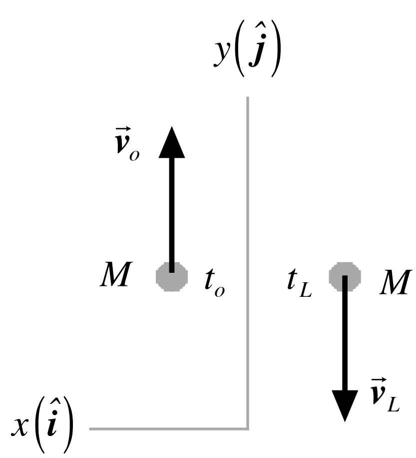
$$\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_L - \vec{v}_o}{\Delta t} = \frac{\left[2.0335\ m/s\ \hat{j}\right] - \left[26.5460\ m/s\ \hat{j}\right]}{(2.500\ s)} = -\ 9.805\frac{m}{s^2}\ \hat{j}\ . \quad (1)$$

$$a_{ave} = 9.805\ m/s^2 \quad and \quad \hat{a}_{ave} = -\ \hat{j}\ . \quad (2)$$

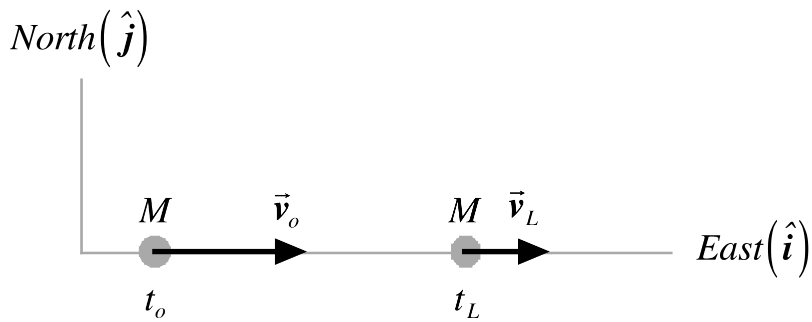
2.) Solution:

$$\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_L - \vec{v}_o}{\Delta t} = \frac{\left[-19.610\ m/s\ \hat{j}\right] - \left[19.610\ m/s\ \hat{j}\right]}{(4.00\ s)} = -\ 9.805\frac{m}{s^2}\ \hat{j}\ . \quad (1)$$

$$a_{ave} = 9.805\ m/s^2 \quad and \quad \hat{a}_{ave} = -\ \hat{j}\ . \quad (2)$$



3.) Solution:



$$\vec{a}_{ave} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_L - \vec{v}_o}{\Delta t} = \frac{\left[2.526\ m/s\ \hat{i}\right] - \left[24.587\ m/s\ \hat{i}\right]}{(3.00\ s)} = -\ 7.354\,\frac{m}{s^2}\,\hat{i}\ . \qquad (1)$$

$$a_{ave} = 7.354\ m/s^2 \quad and \quad \hat{a}_{ave} = -\,\hat{i} \equiv west\ . \qquad (2)$$

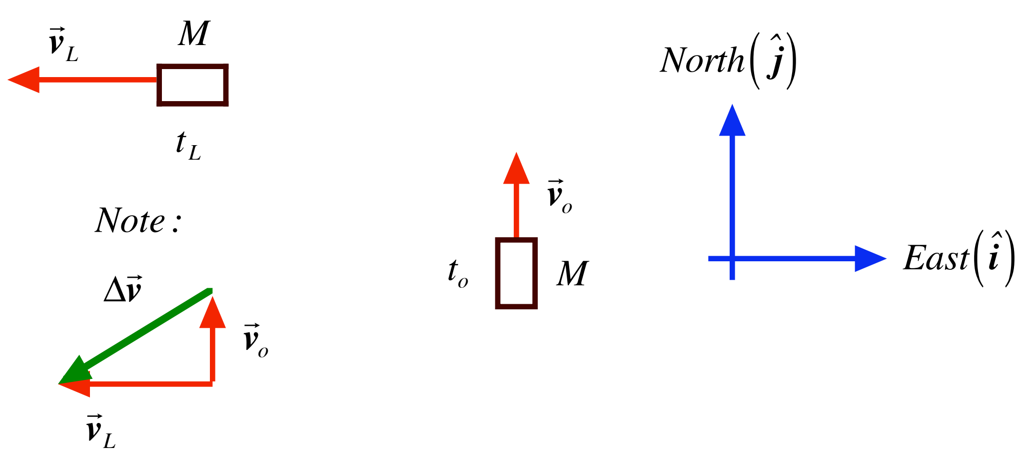
4.) Solution:

$$\vec{a}_{ave} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_L - \vec{v}_o}{\Delta t} = \frac{\left[-33.528\ m/s\ \hat{i}\right] - \left[17.882\ m/s\ \hat{j}\right]}{(6.00\ s)}$$

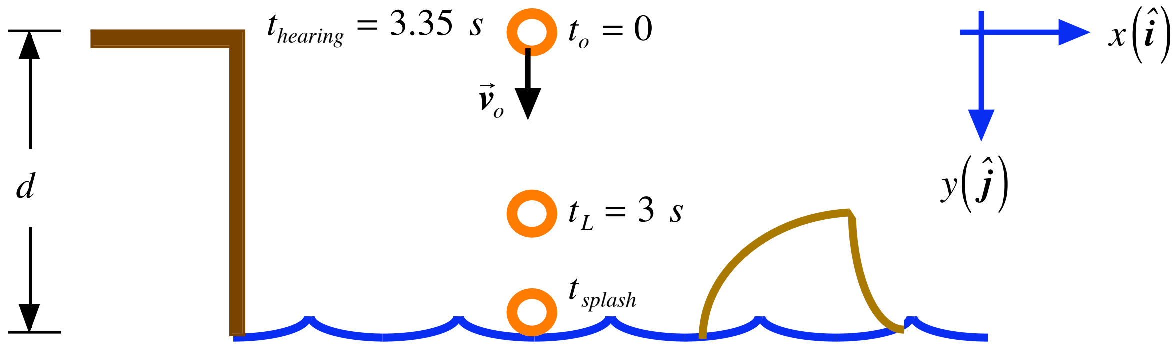
$$= -\ 5.588\,\frac{m}{s^2}\,\hat{i} - 2.980\,\frac{m}{s^2}\,\hat{j}\ . \qquad (1)$$

$$a_{ave} = \sqrt{\left(-5.588\ m/s^2\right)^2 + \left(-2.98\ m/s^2\right)^2} = 6.333\ m/s^2\ . \qquad (2)$$

$$\hat{a}_{ave} = \frac{\vec{a}_{ave}}{a_{ave}} = \frac{-\ 5.588\ m/s^2\ \hat{i} - 2.980\ m/s^2\ \hat{j}}{6.333\ m/s^2} = -\ 0.8824\ \hat{i} - 0.4706\ \hat{j}\ . \ (3)$$



5.) Solution:



$$\vec{a}_{ave} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_L - \vec{v}_o}{\Delta t} = \frac{\left[37.415\ m/s\ \hat{j}\right] - \left[8.000\ m/s\ \hat{j}\right]}{(3.00\ s)} = 9.805\ \frac{m}{s^2}\ \hat{j}\ , \quad (1)$$

$$a_{ave} = 9.805\ m/s^2 \quad and \quad \hat{a}_{ave} = \hat{j} \equiv downward\ . \quad (2)$$

Since sound travels at a constant speed, we can write

$$d = v_{sound}\left(t_{hearing} - t_{splash}\right) = v_{sound} t_{hearing} - v_{sound} t_{splash}\ . \quad (3)$$

The ring is subjected to a constant acceleration and we can write

$$d = v_o t_{splash} + (1/2) a_{ave} t_{splash}^2\ . \quad (4)$$

Equating (3) and (4) gives us for t_{splash} and substituting this into equation (2) gives us

$$v_{sound} t_{hearing} - v_{sound} t_{splash} = v_o t_{splash} + (1/2) a_{ave} t_{splash}^2\ . \quad (5)$$

Collecting terms, we can write

$$(1/2) a_{ave} t_{splash}^2 + \left(v_{sound} + v_o\right) t_{splash} - v_{sound} t_{hearing} = 0\ . \quad (6)$$

Playing algebra games, we find

$$t_{splash}^2 + \frac{2\left(v_{sound} + v_o\right)}{a_{ave}} t_{splash} - \frac{2 v_{sound} t_{hearing}}{a_{ave}}$$

$$= t_{splash}^2 + \frac{2\left((343\ m/s) + (8\ m/s)\right)}{\left(9.805\ m/s^2\right)} t_{splash} - \frac{2(343\ m/s)(3.35\ s)}{\left(9.805\ m/s^2\right)}$$

$$= t_{splash}^2 + [71.5961\ s] t_{splash} - \left[234.438\ s^2\right] = 0\ . \quad (7)$$

Solving this quadratic for t_{splash} we find

$$t_{splash} = -\frac{1}{2}(71.5961\ s) \pm \sqrt{\left[-(1/2)(71.5961\ s)\right]^2 + \left(234.438\ s^2\right)}$$

$$t_{splash} = -35.7981\ s \pm 38.9351\ s\ . \quad (8)$$

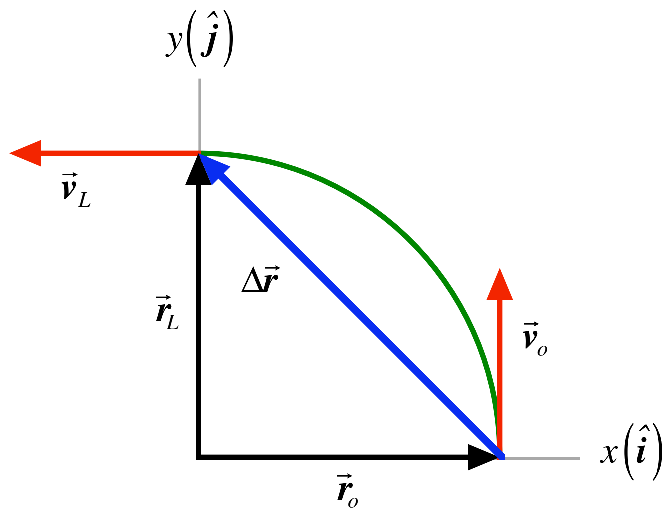
The physically relevant solution is

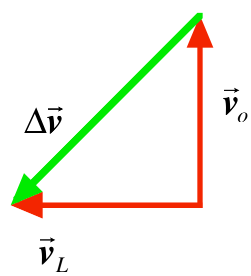
$$t_{splash} = -35.7981\ s + 38.9351\ s = 3.14\ s\ . \tag{9}$$

Finally, using equations (9) and (3) we find

$$d = v_{sound}\left(t_{hearing} - t_{splash}\right) = \left(343\ m/s\right)\left[\left(3.35\ s\right) - \left(3.14\ s\right)\right] = 72.03\ m\ . \tag{10}$$

6.) Solution:





a) $$\Delta\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_L - \vec{\boldsymbol{r}}_o = \left[R\ \hat{\boldsymbol{j}}\right] - \left[R\ \hat{\boldsymbol{i}}\right] = -R\ \hat{\boldsymbol{i}} + R\ \hat{\boldsymbol{j}} = -\left(2.25\ m\right)\hat{\boldsymbol{i}} + \left(2.25\ m\right)\hat{\boldsymbol{j}}\ , \tag{1}$$

$$\Delta r = \sqrt{\left(-R\right)^2 + \left(R\right)^2} = R\sqrt{2} \approx 3.18\ m\ , \tag{2}$$

$$\widehat{\Delta r} = \frac{\Delta\vec{\boldsymbol{r}}}{\Delta r} = \frac{-R\ \hat{\boldsymbol{i}} + R\ \hat{\boldsymbol{j}}}{R\sqrt{2}} = -\frac{1}{\sqrt{2}}\hat{\boldsymbol{i}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{j}}\ . \tag{3}$$

b) $$\vec{\boldsymbol{v}}_{ave} = \frac{\Delta\vec{\boldsymbol{r}}}{\Delta t} = \frac{-\left(2.25\ m\right)\hat{\boldsymbol{i}} + \left(2.25\ m\right)\hat{\boldsymbol{j}}}{\left(0.5437\ s\right)} = -\ 4.138\frac{m}{s}\hat{\boldsymbol{i}} + 4.138\frac{m}{s}\hat{\boldsymbol{j}}\ ,$$

$$v_{ave} = \left(4.138\ m/s\right)\sqrt{2} = 5.852\ m/s\ ,$$

$$\hat{\boldsymbol{v}}_{ave} = \widehat{\Delta r} = -\frac{1}{\sqrt{2}}\hat{\boldsymbol{i}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{j}}\ .$$

c) $$\vec{\boldsymbol{a}}_{ave} = \frac{\Delta\vec{\boldsymbol{v}}}{\Delta t} = \frac{\vec{\boldsymbol{v}}_L - \vec{\boldsymbol{v}}_o}{\Delta t} = \frac{\left(-\ 6.500\ m/s\ \hat{\boldsymbol{i}}\right) - \left(6.500\ m/s\ \hat{\boldsymbol{j}}\right)}{\left(0.5437\ s\right)}$$

$$= -11.955\ m/s^2\ \hat{\boldsymbol{i}} - 11.955\ m/s^2\ \hat{\boldsymbol{j}}\ ,$$

$$a_{ave} = \left(11.955\ m/s^2\right)\sqrt{2} = 16.907\ m/s^2\ .$$

$$\hat{\boldsymbol{a}}_{ave} = \frac{\vec{\boldsymbol{a}}_{ave}}{a_{ave}} = \frac{-11.955\ m/s^2\ \hat{\boldsymbol{i}} - 11.955\ m/s^2\ \hat{\boldsymbol{j}}}{16.907\ m/s^2} = -\ 0.7071\ \hat{\boldsymbol{i}} - 0.7071\ \hat{\boldsymbol{j}}\ .$$

d) Since the speed is constant,

$$\tau = \frac{2\pi R}{v} = \frac{2\pi(2.250\ m)}{(6.500\ m/s)} = 2.175\ s\ .$$

e) $$\Delta\ell = R(\Delta\varphi) = (2.250\ m)((\pi/2)\ radians) = 3.5343\ m\ .$$

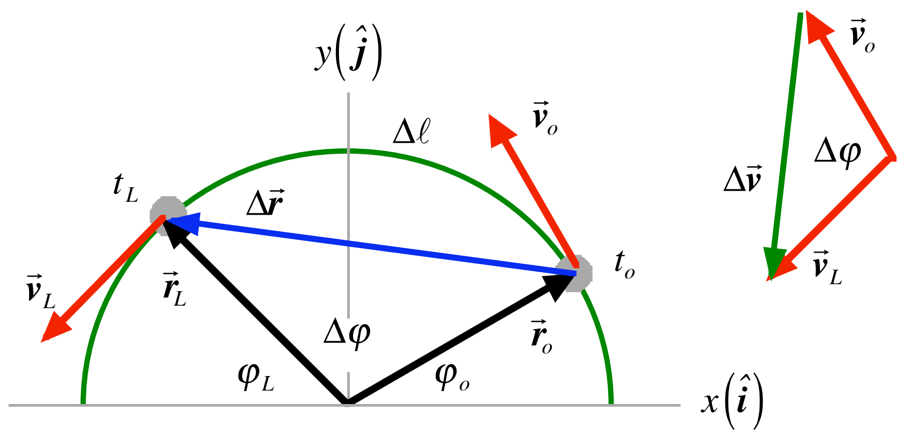
Note that:

$$s_{ave} = \frac{\Delta\ell}{\Delta t} = \frac{(3.5343\ m)}{(0.5437\ s)} = 6.500\ \frac{m}{s} = v\ .$$

We know for circular motion that the instantaneous radial acceleration is

$$a_{rad} = \frac{v^2}{R} = \frac{(6.500\ m/s)^2}{(2.250\ m)} = 18.778\ \frac{m}{s^2}\ .$$

7.) Solution:



a) $$\Delta\vec{r} = \vec{r}_L - \vec{r}_o = \left[-R\cos\varphi_L\ \hat{i} + R\sin\varphi_L\ \hat{j}\right] - \left[R\cos\varphi_o\ \hat{i} + R\sin\varphi_o\ \hat{j}\right]$$

$$= -R(\cos\varphi_L + \cos\varphi_o)\ \hat{i} + R(\sin\varphi_L - \sin\varphi_o)\ \hat{j}$$

$$= -(10\ m)(\cos 45° + \cos 30°)\ \hat{i} + (10\ m)(\sin 45° - \sin 30°)\ \hat{j}$$

$$= -15.7313\ m\ \hat{i} + 2.0711\ m\ \hat{j}\ .$$

$$\Delta r = \sqrt{(-15.7313\ m)^2 + (2.0711\ m)^2} = 15.8670\ m\ ,$$

$$\hat{\Delta r} = \frac{\Delta\vec{r}}{\Delta r} = \frac{-15.7313\ m\ \hat{i} + 2.0711\ m\ \hat{j}}{15.8670\ m} = -\ 0.9914\ \hat{i} + 0.1305\ \hat{j}\ .$$

b) $$\vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t} = \frac{-15.7313\ m\ \hat{i} + 2.0711\ m\ \hat{j}}{(3.0000\ s)} = -\ 5.244\ \frac{m}{s}\ \hat{i} + 0.690\ \frac{m}{s}\ \hat{j}\ ,$$

$$v_{ave} = \sqrt{(-\ 5.244\ m/s)^2 + (0.690\ m/s)^2} = 5.289\ m/s\ ,$$

$$\hat{v}_{ave} = \hat{\Delta r} = -\ 0.9914\ \hat{i} + 0.1305\ \hat{j}\ .$$

d) $$\Delta\ell = R(\Delta\varphi) = (10\ m)(105(\pi/180)\ radians) = 18.326\ m\ .$$

e) As the speed is constant, we have

$$v = s_{ave} = \frac{\Delta\ell}{\Delta t} = \frac{18.326\ m}{(3\ s)} = 6.109\ m/s\ .$$

c) $$\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_L - \vec{v}_o}{\Delta t} = \frac{\left[-v\cos 45°\ \hat{i} - v\sin 45°\ \hat{j}\right] - \left[-v\cos 60°\ \hat{i} + v\sin 60°\ \hat{j}\right]}{\Delta t}$$

$$= \frac{-v(\cos 45° - \cos 60°)\ \hat{i} - v(\sin 45° + \sin 60°)\ \hat{j}}{\Delta t}$$

$$= \left[-(6.109\ m/s)(\cos 45° - \cos 60°)\ \hat{i} - (6.109\ m/s)(\sin 45° + \sin 60°)\ \hat{j}\right]/3\ s$$

$$= -\,0.422\ m/s^2\ \hat{i} - 3.203\ m/s^2\ \hat{j}\ ,$$

$$a_{ave} = \sqrt{(-\,0.422\ m/s^2)^2 + (-\,3.203\ m/s^2)^2} = 3.231\ m/s^2\ .$$

$$\hat{a}_{ave} = \frac{\vec{a}_{ave}}{a_{ave}} = \frac{-\,0.422\ m/s^2\ \hat{i} - 3.203\ m/s^2\ \hat{j}}{3.231\ m/s^2} = -\,0.1306\ \hat{i} - 0.9913\ \hat{j}\ .$$

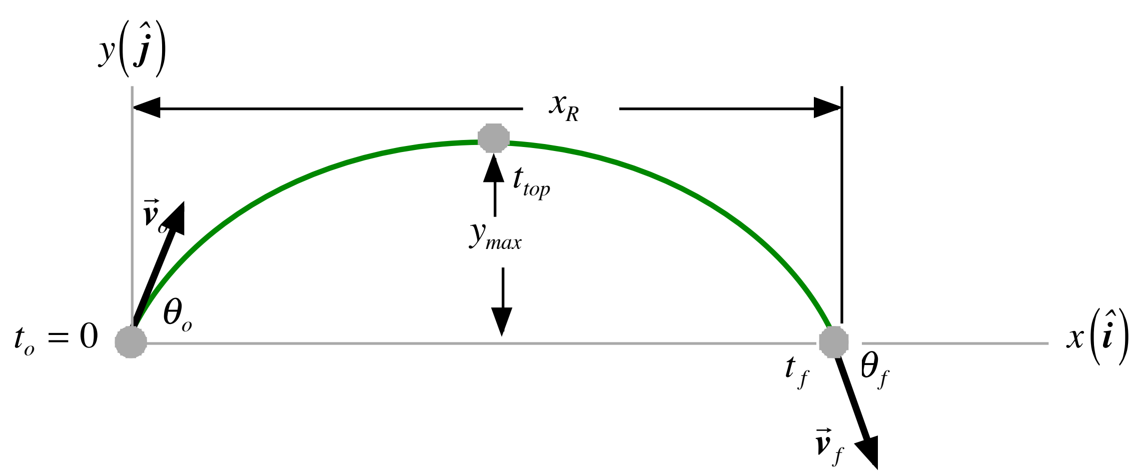
f) Since the speed is constant,

$$\tau = \frac{2\pi R}{v} = \frac{2\pi(10\ m)}{(6.109\ m/s)} = 10.285\ s\ .$$

Note: For circular motion, the instantaneous radial acceleration is given by

$$a_{rad} = \frac{v^2}{R} = \frac{(6.109\ m/s)^2}{(10\ m)} = 3.732\ \frac{m}{s^2}\ .$$

8.) Solution:



The equations of motion are:

$$x = v_o\,cos(\theta_o)t\ , \tag{1}$$

$$y = v_o\,sin(\theta_o)t - (1/2)gt^2\ , \tag{2}$$

$$v_x = v_o\,cos(\theta_o)\ , \tag{3}$$

$$v_y = v_o\,sin(\theta_o) - gt\ . \tag{4}$$

a) At its maximum height, the y component of the velocity is zero. So, from equation (4),

$$v_y = 0 = v_o\ sin(\theta_o) - gt_{top}\ , \tag{5}$$

and

$$t_{top} = v_o\ sin(\theta_o)/g\ . \tag{6}$$

Equation (6) into equation (2) yields

$$y_{max} = v_o\ sin(\theta_o)(v_o\ sin(\theta_o)/g) - (1/2)g(v_o\ sin(\theta_o)/g)^2$$

$$= \frac{(v_o\,sin(\theta_o))^2}{2g} = \frac{((25\ m/s)(sin\,35°))^2}{2(9.805\ m/s^2)} = 10.49\ m\ . \tag{7}$$

b) The y coordinate at the instant the object strikes the ground is zero and the time is t_f. Using equation (2), we have

$$0 = v_o\ sin(\theta_o)t_f - (1/2)gt_f^2\ , \tag{8}$$

$$v_o\ sin(\theta_o)t_f = (1/2)gt_f^2\ , \tag{9}$$

$$t_f = \frac{2v_o\ sin(\theta_o)}{g} = \frac{2(25\ m/s)(sin\,35°)}{(9.805\ m/s^2)} = 2.925\ s\ . \tag{10}$$

So,

$$x_R = v_o\,cos(\theta_o)t_f = (25\ m/s)(cos\,35°)(2.925\ s) = 59.90\ m\ . \tag{11}$$

c) The x component of the velocity as the object strikes the ground is

$$v_{x,f} = v_{x,o} = v_o\,cos(\theta_o)$$

$$= (25\ m/s)(cos\,35°) = 20.48\ m/s\ . \tag{12}$$

The y component of the velocity as the object strikes the ground is

$$v_{y,f} = v_o\ sin(\theta_o) - gt_f$$

$$= (25\ m/s)(sin\,35°) - (9.805\ m/s^2)(2.925\ s) = -14.34\ m/s\ . \tag{13}$$

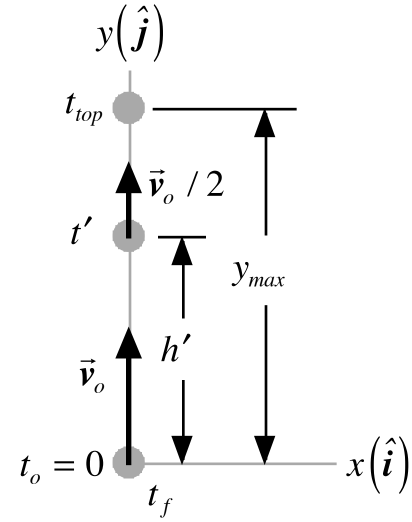
So, the final speed is given by

$$v_f = \sqrt{(20.48\ m/s)^2 + (-14.34\ m/s)^2} = 25.00\ m/s\ . \tag{14}$$

d)

$$\theta_f = tan^{-1}\left[\frac{|v_{y,f}|}{|v_{x,f}|}\right] = tan^{-1}\left[\frac{|-14.34\ m/s|}{|20.48\ m/s|}\right] = 35°\ . \tag{15}$$

9.) Solution:



The equations of motion are:

$$y = v_o t - (1/2) g t^2 \ , \tag{1}$$

$$v_y = v_o - g t \ . \tag{2}$$

a) At the top, the instantaneous speed is zero and the time is t_{top}. From equation (2), we write

$$0 = v_o - g t_{top} \ , \tag{3}$$

and

$$t_{top} = \frac{v_o}{g} = \frac{(15 \ m/s)}{(9.805 \ m/s^2)} = 1.530 \ s \ . \tag{4}$$

Substitution of equation (4) into equation (1) gives us the maximum height.

$$y_{max} = v_o t_{top} - (1/2) g t_{top}^2$$

$$= (15 \ m/s)(1.530 \ s) - (1/2)(9.805 \ m/s^2)(1.530 \ s)^2 = 11.47 \ m \ . \tag{5}$$

b) To find the height at which the speed is half of its initial speed, we use equation (2) to note that

$$v_o / 2 = v_o - g t' , \tag{6}$$

and, therefore,

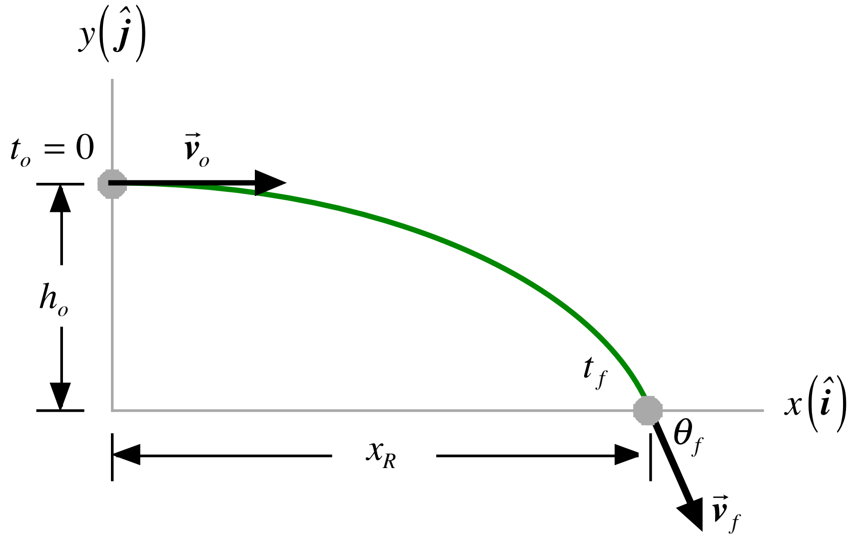
$$t' = \frac{v_o}{2g} = \frac{(15 \ m/s)}{2(9.805 \ m/s^2)} = 0.765 \ s \ . \tag{7}$$

So, we have from equations (7) and (1),

$$h' = v_o t' - (1/2) g t'^2$$

$$= (15 \ m/s)(0.765 \ s) - (1/2)(9.805 \ m/s^2)(0.765 \ s)^2 = 8.61 \ m \ . \tag{8}$$

10.) Solution:



The equations of motion are:

$$x = v_o t \ , \tag{1}$$

$$y = h_o - (1/2) g t^2 \ , \tag{2}$$

$$v_x = v_o \ , \tag{3}$$

$$v_y = -gt \ . \tag{4}$$

a) The y coordinate at the instant the object strikes the ground is zero and the time is t_f. Using equation (2), we have

$$0 = h_o - (1/2) g t_f^2 \ , \tag{5}$$

$$t_f = \sqrt{2h_o / g} = \sqrt{2(22.5 \ m)/(9.805 \ m/s^2)} = 2.142 \ s \ . \tag{6}$$

So,
$$x_R = v_o t_f = (45.75 \ m/s)(2.142 \ s) = 98.01 \ m \ . \tag{7}$$

b) The x component of the velocity as the object strikes the ground is

$$v_{x,f} = v_{x,o} = v_o = 45.75 \ m/s \ . \tag{8}$$

The y component of the velocity as the object strikes the ground is

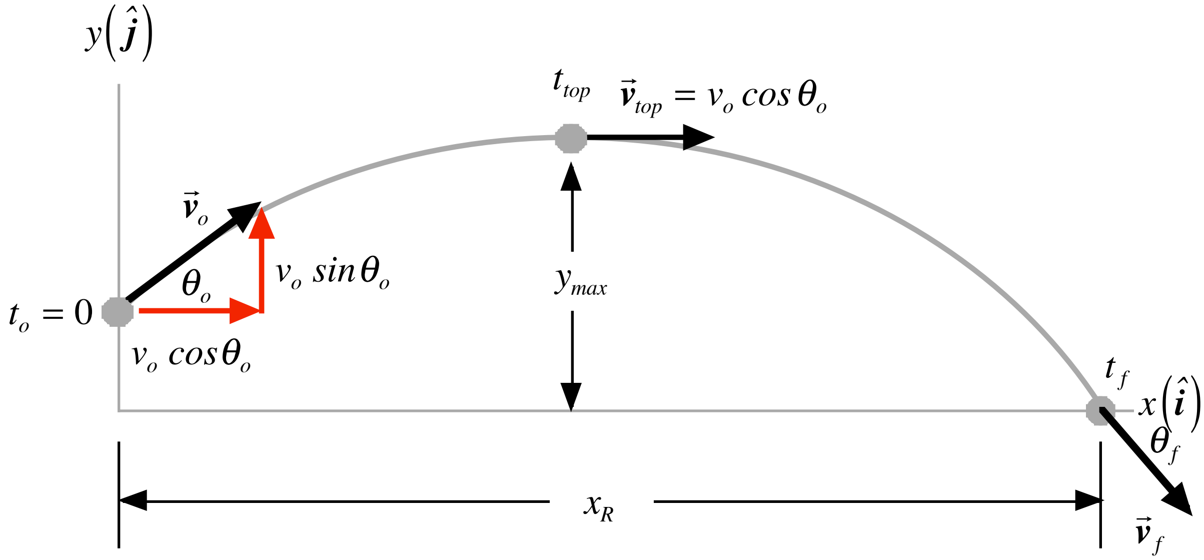
$$v_{y,f} = -g t_f$$
$$= -(9.805 \ m/s^2)(2.142 \ s) = - \ 21.00 \ m/s \ . \tag{9}$$

So, the final speed is given by

$$v_f = \sqrt{(45.75 \ m/s)^2 + (-21.00 \ m/s)^2} = 50.34 \ m/s \ . \tag{10}$$

$$\theta_f = tan^{-1}\left[\frac{|v_{y,f}|}{|v_{x,f}|}\right] = tan^{-1}\left[\frac{|-21.00 \ m/s|}{|45.75 \ m/s|}\right] = 24.66° \ . \tag{11}$$

11.) Solution:



The equations of motion are:

$$x = v_o \cos(\theta_o) t \ , \tag{1}$$

$$y = h_o + v_o \sin(\theta_o) t - (1/2) g t^2 \ , \tag{2}$$

$$v_x = v_o \cos(\theta_o) \ , \tag{3}$$

$$v_y = v_o \sin(\theta_o) - gt \ . \tag{4}$$

a) At its maximum height, the y component of the velocity is zero. So, from equation (4),

$$v_y = 0 = v_o \sin(\theta_o) - g t_{top} \ , \tag{5}$$

and

$$t_{top} = v_o \sin(\theta_o) / g \ . \tag{6}$$

Equation (6) into equation (2) yields

$$y_{max} = h_o + v_o \sin(\theta_o)\left(v_o \sin(\theta_o)/g\right) - (1/2) g \left(v_o \sin(\theta_o)/g\right)^2$$

$$= h_o + \frac{\left(v_o \sin(\theta_o)\right)^2}{2g} = (10 \ m) + \frac{\left((25 \ m/s)(\sin 35°)\right)^2}{2\left(9.805 \ m/s^2\right)} = 20.49 \ m \ . \tag{7}$$

b) The y coordinate at the instant the object strikes the ground is zero and the time is t_f. Using equation (2), we have

$$0 = h_o + v_o \sin(\theta_o) t_f - (1/2) g t_f^2 \ , \tag{8}$$

$$t_f^2 - \frac{2 v_o \sin(\theta_o)}{g} t_f - \frac{2 h_o}{g} = 0 \ , \tag{9}$$

$$t_f = \frac{v_o \sin(\theta_o)}{g} + \sqrt{\left[\frac{v_o \sin(\theta_o)}{g}\right]^2 + \frac{2h_o}{g}}$$

$$= \frac{(25 m/s)(\sin 35°)}{(9.805 m/s^2)} + \sqrt{\left[\frac{(25 m/s)(\sin 35°)}{(9.805 m/s^2)}\right]^2 + \left[\frac{2(10 m)}{(9.805 m/s^2)}\right]}$$

$$= (1.46246\ s) + \sqrt{(1.46246\ s)^2 + (2.03978\ s^2)} = 3.507\ s\ . \qquad (10)$$

So, $$x_R = v_o \cos(\theta_o) t_f = (25\ m/s)(\cos 35°)(3.507\ s) = 71.82\ m\ . \qquad (11)$$

c) The x component of the velocity as the object strikes the ground is

$$v_{x,f} = v_{x,o} = v_o \cos(\theta_o)$$

$$= (25\ m/s)(\cos 35°) = 20.48\ m/s\ . \qquad (12)$$

The y component of the velocity as the object strikes the ground is

$$v_{y,f} = v_o \sin(\theta_o) - g t_f$$

$$= (25\ m/s)(\sin 35°) - (9.805\ m/s^2)(3.507\ s) = -20.05\ m/s\ . \qquad (13)$$

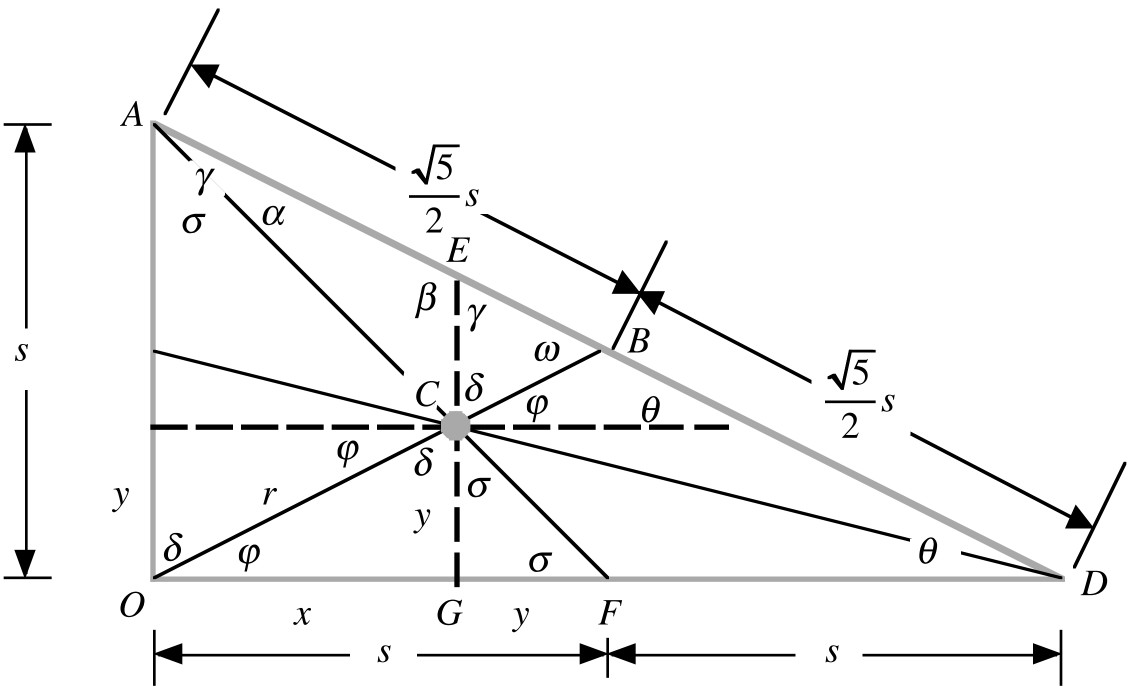
So, the final speed is given by

$$v_f = \sqrt{(20.48\ m/s)^2 + (-20.05\ m/s)^2} = 28.66\ m/s\ . \qquad (14)$$

d) $$\theta_f = \tan^{-1}\left[\frac{|v_{y,f}|}{|v_{x,f}|}\right] = \tan^{-1}\left[\frac{|-20.05\ m/s|}{|20.48\ m/s|}\right] = 44.4°\ . \qquad (15)$$

Solutions to Problems for Chapter 8

1.) Solution:



Note first that point C represents the center of mass. Let us list some things we can learn from the diagram:

$$tan\theta = \left[\frac{s}{2s}\right] \rightarrow \theta = tan^{-1}\left[0.50\right] = 25.56505° \qquad (1)$$

$$\gamma = 90° - \theta = 63.43495° \qquad (2)$$

$$\sigma = 45° \qquad (3)$$

$$\beta = 180° - \gamma = 116.56505° \qquad (4)$$

Inspection of right triangle ΔOCG suggests that if we knew either angle δ or φ we could easily calculate x and y. To that end, we use the law of sines with ΔOAB. We can write

$$\frac{\left(\sqrt{5}/2\right)s}{sin\,\delta} = \frac{s}{sin\,\omega} = \frac{s}{sin\left(180° - \gamma - \delta\right)} = \frac{s}{sin\left(\beta - \delta\right)}\ , \qquad (5)$$

and

$$\frac{sin\left(\beta - \delta\right)}{sin\,\delta} = \frac{2}{\sqrt{5}}\ . \qquad (6)$$

Using the trig identity

$$sin\left(\beta - \delta\right) = sin\,\beta\,cos\,\delta - cos\,\beta\,sin\,\delta\ , \qquad (7)$$

we can write

$$\frac{sin\,\beta\,cos\,\delta - cos\,\beta\,sin\,\delta}{sin\,\delta} = \frac{sin\,\beta}{tan\,\delta} - cos\,\beta = \frac{2}{\sqrt{5}}\ . \qquad (8)$$

rearranging terms, we have

$$\frac{1}{tan\,\delta} = \left[\frac{2}{\sqrt{5}} + cos\,\beta\right] / \, sin\,\beta \;, \tag{9}$$

and

$$tan\,\delta = \frac{sin\,\beta}{\left(2/\sqrt{5}\right) + cos\,\beta} \;, \tag{10}$$

with, at last,

$$\delta = tan^{-1}\left[\frac{sin\,\beta}{\left(2/\sqrt{5}\right) + cos\,\beta}\right] = tan^{-1}\left[\frac{sin\left(116.56505°\right)}{\left(2/\sqrt{5}\right) + cos\left(116.56505°\right)}\right]$$

$$= tan^{-1}\left[\frac{sin\left(116.56505°\right)}{\left(2/\sqrt{5}\right) + cos\left(116.56505°\right)}\right] = 63.43495° = \gamma \,. \tag{11}$$

Now, using the law of sines with ΔOCF, we can write

$$\frac{r}{sin\,45°} = \frac{s}{sin\left(\delta + \sigma\right)} = \frac{s}{sin\left(108.43495°\right)} \;, \tag{12}$$

and

$$r = \left[\frac{sin\,45°}{sin\left(108.43495°\right)}\right] s = \left(0.745356\right) s \,. \tag{13}$$

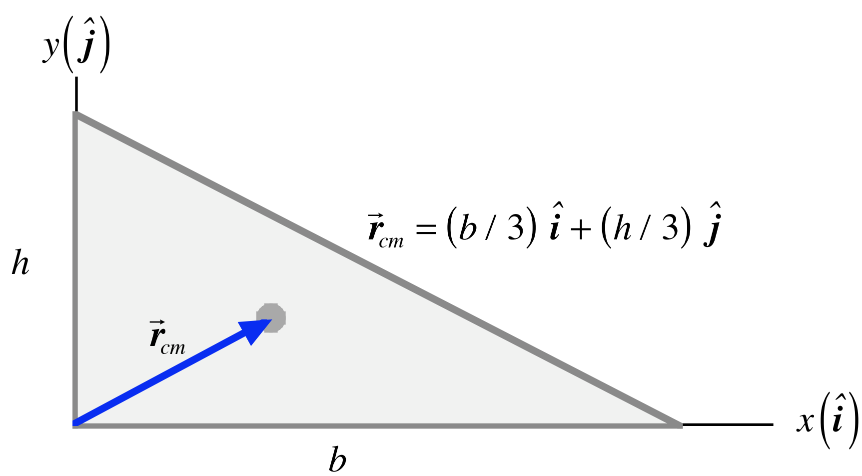
Therefore,

$$x = r\,sin\,\delta = \left[\left(0.745356\right)s\right]\left[sin\,63.43495°\right] = \left(2/3\right)s \,, \tag{14}$$

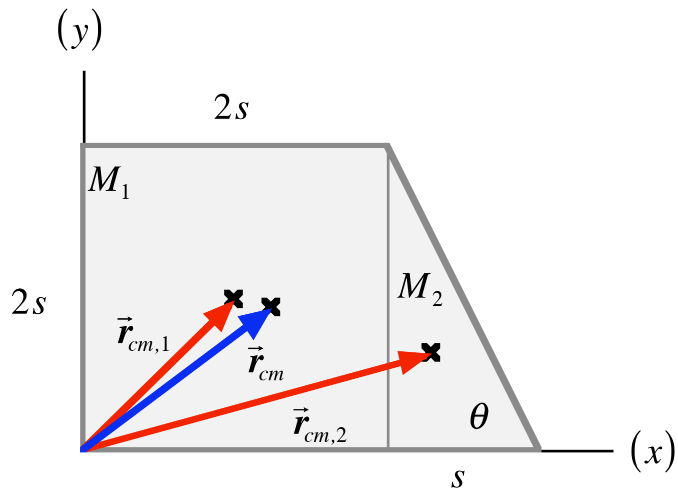
and

$$y = r\,cos\,\delta = \left[\left(0.745356\right)s\right]\left[cos\,63.43495°\right] = \left(1/3\right)s \,. \tag{15}$$

This is, of course, not to be confused with a proof for all triangles. For that, we would use the calculus. We can, however, generalize our results for right triangles; as represented in the diagram below:



2.)   Solution:



First, we divide the object into a square, of mass M_1 and a triangle of mass M_2. So, the total mass of the original object is

$$M = M_1 + M_2 \ . \tag{1}$$

We can write the center of mass of each object as follows:

$$\vec{r}_{cm,1} = s\ \hat{i} + s\ \hat{j}\ , \tag{2}$$

$$\vec{r}_{cm,2} = (7/3)s\ \hat{i} + (2/3)s\ \hat{j}\ . \tag{3}$$

As the material is homogeneous, we define the average surface mass density σ_{ave} by

$$\sigma_{ave} = \frac{M}{A_{tot}} = \frac{M}{(2s)^2 + (1/2)(s)(2s)} = \frac{M}{5s^2}\ . \tag{4}$$

We can now use this to calculate the mass of the square and the triangle. We have

$$M_1 = \sigma_{ave} A_1 = \left[\frac{M}{5s^2}\right]\left[4s^2\right] = \frac{4}{5}M\ , \tag{5}$$

$$M_2 = \sigma_{ave} A_2 = \left[\frac{M}{5s^2}\right]\left[s^2\right] = \frac{1}{5}M\ . \tag{6}$$

We are now in a position to calculate the center of mass. Recall that we use

$$\vec{r}_{cm} = \frac{\sum_{i=1}^{N} M_i \vec{r}_{cm,i}}{\sum_{i=1}^{N} M_i} = \frac{1}{M_{tot}} \sum_{i=1}^{N} M_i \vec{r}_{cm,i}\ . \tag{7}$$

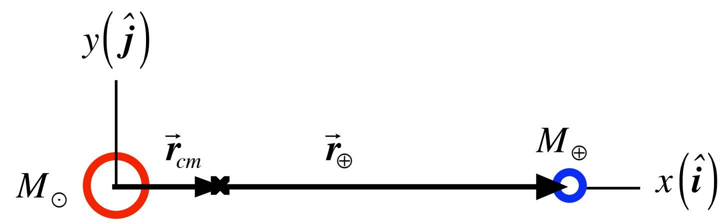
We write

$$\vec{r}_{cm} = \frac{1}{M}\left[M_1 \vec{r}_{cm,1} + M_2 \vec{r}_{cm,2}\right]$$

$$= \frac{1}{M}\left[((4/5)M)\left(s\ \hat{i} + s\ \hat{j}\right) + ((1/5)M)\left((7/3)s\ \hat{i} + (2/3)s\ \hat{j}\right)\right]$$

$$= \frac{(19/15)Ms\ \hat{\boldsymbol{i}} + (14/15)Ms\ \hat{\boldsymbol{j}}}{M} = \frac{19}{15}s\ \hat{\boldsymbol{i}} + \frac{14}{15}s\ \hat{\boldsymbol{j}}\ .$$

3.) Solution:



We can write,

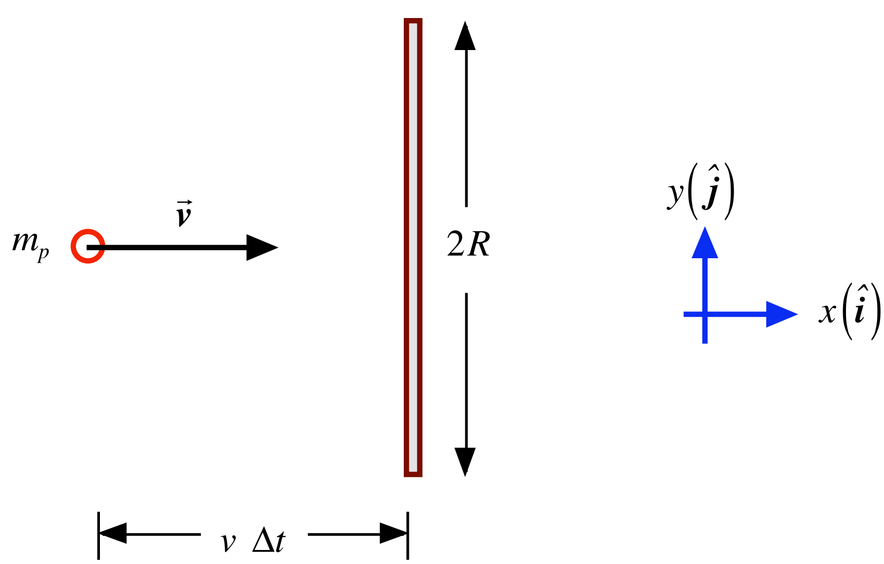
$$\vec{\boldsymbol{r}}_{cm} = \frac{1}{M_{tot}}\sum_{i=1}^{N} M_i \vec{\boldsymbol{r}}_{cm,i} = \frac{M_{\odot}\vec{\boldsymbol{r}}_{cm,\odot} + M_{\oplus}\vec{\boldsymbol{r}}_{cm,\oplus}}{M_{\odot} + M_{\oplus}}$$

$$= \frac{\left(1.99\times10^{30}\,kg\right)(0) + \left(5.98\times10^{24}\,kg\right)\left(1.496\times10^{11}\,m\ \hat{\boldsymbol{i}}\right)}{\left(1.99\times10^{30}\,kg\right) + \left(5.98\times10^{24}\,kg\right)}$$

$$= 4.50\times10^{5}\ \ m\ \hat{\boldsymbol{i}} = \vec{\boldsymbol{r}}_{cm}\,.$$

As the radius of the Sun is $R_{\odot} = 6.955\times10^{8}\,m$, the barycenter of the Sun-Earth system is actually at a point interior to the Sun.

4.) Solution:



A proton that strikes the satellite will either be absorbed or deflected. We will assume that an individual proton only sees a planar, circular cross-section of radius R. We also assume that all protons are reflected and suffer an elastic collision. For an individual proton, the change in linear momentum would be

$$\Delta\vec{\boldsymbol{p}}_{one\ \ proton} = \vec{\boldsymbol{p}}_L - \vec{\boldsymbol{p}}_o = \left[-m_p v\ \hat{\boldsymbol{i}}\right] - \left[-m_p v\ \hat{\boldsymbol{i}}\right] = -2m_p v\ \hat{\boldsymbol{i}}\ . \qquad (1)$$

In a time interval Δt, the number of protons striking the satellite would be given by

$$N_p = n_p V_{olume} = n_p \left(v \ \Delta t \ A_{rea} \right). \qquad (2)$$

So, the average force exerted on the protons by the satellite would be

$$\vec{\boldsymbol{F}}_{ave,\ on\ protons} = N_p \frac{\Delta \vec{\boldsymbol{p}}_{one\ proton}}{\Delta t} = \frac{\left(n_p v \ \Delta t \ A_{rea}\right)\left(-2 m_p v \ \hat{\boldsymbol{i}}\right)}{\Delta t} = -2 n_p m_p v^2 A_{rea} \ \hat{\boldsymbol{i}} \ , \qquad (3)$$

while the average force exerted on the satellite by the protons would be same magnitude and opposite direction. We have

$$\vec{\boldsymbol{F}}_{ave,\ on\ satellite} = 2\left(10^7 / m^3\right)\left(1.673 \times 10^{-27} kg\right)\left(4.0 \times 10^5 \ m/s\right)^2 \left(1 \ m^2\right) \hat{\boldsymbol{i}}$$

$$= 5.35 \times 10^{-9} \ kg \ m / s^2 \ \hat{\boldsymbol{i}} \ .$$

(This is merely an approximation!)

Solutions to Problems for Chapters 9 and 10

1.) Solution:

a) The magnitude of the average force exerted on the Sun by the Earth is given by

$$F^G_{\odot\oplus} = \frac{GM_\odot M_\oplus}{r^2_{\odot\oplus}}$$

$$= \frac{(6.67\times10^{-11}\,Nm^2/kg^2)(1.99\times10^{30}\,kg)(5.97\times10^{24}\,kg)}{(1.496\times10^{11}\,m)^2}$$

$$= 3.54\times10^{22}\;N\;. \quad (1)$$

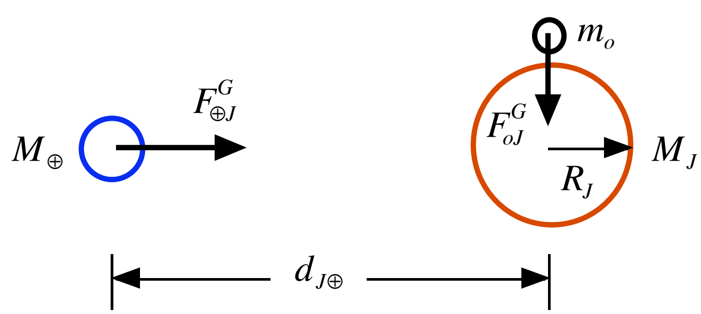
b) The magnitude of he gravitational force exerted by the Sun on an object close to the surface of the Sun is given by

$$F^G_{o\odot} = \frac{Gm_oM_\odot}{R^2_\odot} = m_o\left[\frac{GM_\odot}{R^2_\odot}\right] = m_oa_o\;, \quad (2)$$

and the magnitude of the acceleration of an object close to the surface of the Sun is

$$a_o = \frac{(6.67\times10^{-11}Nm^2/kg^2)(1.99\times10^{30}\;kg)}{(6.955\times10^8\,m)^2} = 274\frac{m}{s^2} \approx 28\;g\;. \quad (3)$$

2.) Solution:



a) The magnitude of the force exerted on the Earth by Jupiter is given by

$$F^G_{\oplus J} = \frac{GM_\oplus M_J}{d^2_{J\oplus}}$$

$$= \frac{(6.67\times10^{-11}\,Nm^2/kg^2)(5.97\times10^{24}\;kg)(1.90\times10^{27}\;kg)}{(6.29\times10^{11}\,m)^2}$$

$$= 1.91\times10^{18}\;N\;. \quad (1)$$

b) The magnitude of the force exerted on an object near the surface of Jupiter is given by

$$F^G_{oJ} = \frac{Gm_oM_J}{R^2_J} = m_o\left[\frac{GM_J}{R^2_J}\right] = m_oa_o\;, \quad (2)$$

and the acceleration of this object is

$$a_o = \frac{GM_J}{R_J^2}$$

$$= \frac{\left(6.67\times10^{-11}\,Nm^2/kg^2\right)\left(1.90\times10^{27}\ kg\right)}{\left(7.137\times10^{7}\ m\right)^2} = 24.9\,\frac{m}{s^2} \approx 2.54\,g\ . \qquad (3)$$

3.) Solution:

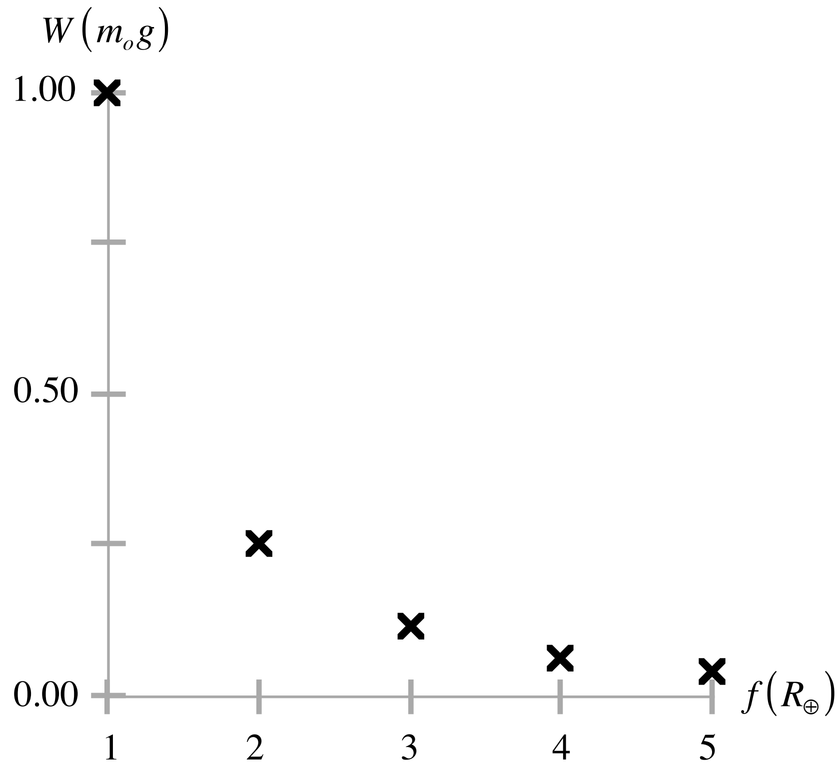
a)

$$F_{o\oplus}^G = \frac{Gm_oM_\oplus}{d^2} = \frac{Gm_oM_\oplus}{\left(fR_\oplus\right)^2} = \frac{1}{f^2}\left[m_o\left(\frac{GM_\oplus}{R_\oplus^2}\right)\right] = \frac{m_og}{f^2} = \frac{W}{f^2}\ , \qquad (1)$$

where W is the weight of the object at the surface of the Earth and is given by

$$W = m_o\left[\frac{GM_\oplus}{R_\oplus^2}\right] = m_og\ . \qquad (2)$$

b)



c) Using Newton's second law,

$$F_{o\oplus}^G = \frac{1}{f^2}\left[\frac{Gm_oM_\oplus}{R_\oplus^2}\right] = m_oa_o\ , \qquad (3)$$

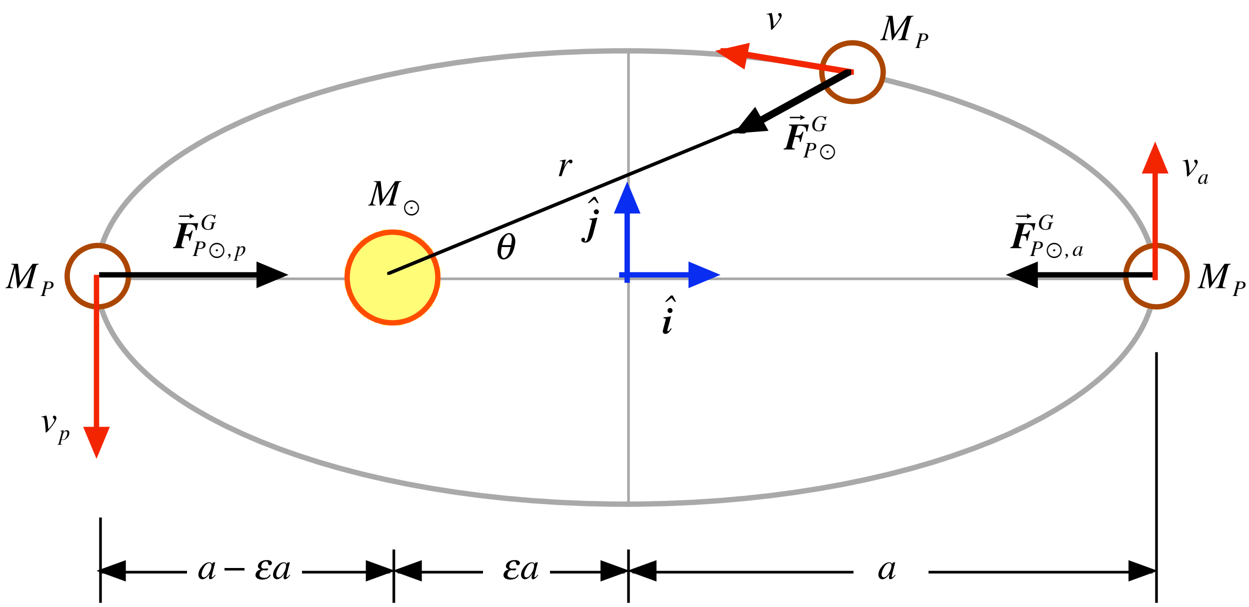
and

$$a_o = \frac{1}{f^2}\left[\frac{GM_\oplus}{R_\oplus^2}\right] = \frac{g}{f^2}\ . \qquad (4)$$

d) For objects at the surface of the Earth, $f = 1$. If $f = 3$, our object is at a distance from the center of the Earth equal to three Earth radii and the acceleration becomes

$$a_o = \frac{g}{f^2} = \frac{9.789\ m/s^2}{(3)^2} = 1.088\ \frac{m}{s^2}. \qquad (5)$$

4.) Solution:



a) (i)

$$\vec{F}^G_{P\odot} = \frac{GM_PM_\odot}{r^2}\left[-\cos\theta\ \hat{i} - \sin\theta\ \hat{j}\right]$$

(ii)

$$\vec{F}^G_{P\odot,p} = \frac{GM_PM_\odot}{\left[a(1-\varepsilon)\right]^2}\ \hat{i} = \frac{1}{(1-\varepsilon)^2}\left[\frac{GM_PM_\odot}{a^2}\right]\hat{i}\ . \qquad (1)$$

(iii)

$$\vec{F}^G_{P\odot,a} = -\frac{GM_PM_\odot}{\left[a(1+\varepsilon)\right]^2}\ \hat{i} = -\frac{1}{(1+\varepsilon)^2}\left[\frac{GM_PM_\odot}{a^2}\right]\hat{i}\ . \qquad (2)$$

b) (i)

$$F^G_{P\odot,p} = \frac{1}{(1-\varepsilon)^2}\left[\frac{GM_PM_\odot}{a^2}\right] = \frac{M_Pv^2_{P,p}}{(1-\varepsilon)a}\ , \qquad (3)$$

$$v_{P,p} = \sqrt{\frac{GM_\odot}{(1-\varepsilon)a}}\ . \qquad (4)$$

(ii)

$$F^G_{P\odot,a} = \frac{1}{(1+\varepsilon)^2}\left[\frac{GM_PM_\odot}{a^2}\right] = \frac{M_Pv^2_{P,a}}{(1+\varepsilon)a}\ , \qquad (5)$$

$$v_{P,a} = \sqrt{\frac{GM_\odot}{(1+\varepsilon)a}} \ . \tag{6}$$

c) (i)

$$v_{\oplus,p} = \sqrt{\frac{GM_\odot}{(1-\varepsilon_\oplus)a_\oplus}} = \sqrt{\frac{\left(6.67\times10^{-11}\,Nm^2/kg^2\right)\left(1.99\times10^{30}\,kg\right)}{\left(1-(.0167)\right)\left(1.496\times10^{11}\,m\right)}}$$

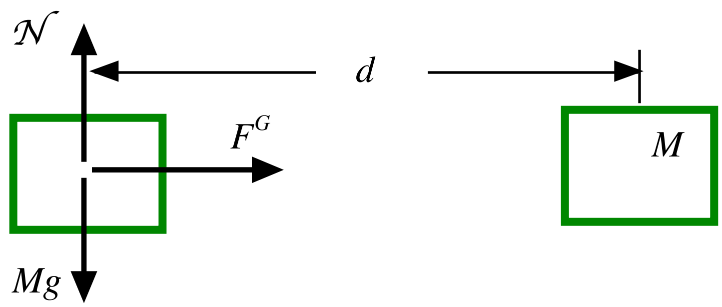
$$= 3.00\times10^4\ \frac{m}{s} \approx 67{,}110\ \ mph\ . \tag{7}$$

(ii)

$$v_{\oplus,a} = \sqrt{\frac{GM_\odot}{(1+\varepsilon_\oplus)a_\oplus}} = \sqrt{\frac{\left(6.67\times10^{-11}\,Nm^2/kg^2\right)\left(1.99\times10^{30}\,kg\right)}{\left(1+(.0167)\right)\left(1.496\times10^{11}\,m\right)}}$$

$$= 2.95\times10^4\ \frac{m}{s} \approx 65{,}990\ \ mph\ . \tag{8}$$

5.) Solution: a)



We can write

$$F^G = \frac{G(M)(M)}{d^2} = Ma\ . \tag{1}$$

and, therefore,

$$a = \frac{GM}{d^2}\ , \tag{2}$$

Initially, when $d = 1\ \ m$, we have

$$a_o = \frac{\left(6.67\times10^{-11}\,Nm^2/C^2\right)\left(100\ \ kg\right)}{\left(1\ \ m\right)^2} = 6.67\times10^{-9}\ \frac{m}{s^2}\ . \tag{3}$$

b) When we apply what we know about static friction to this situation, we get

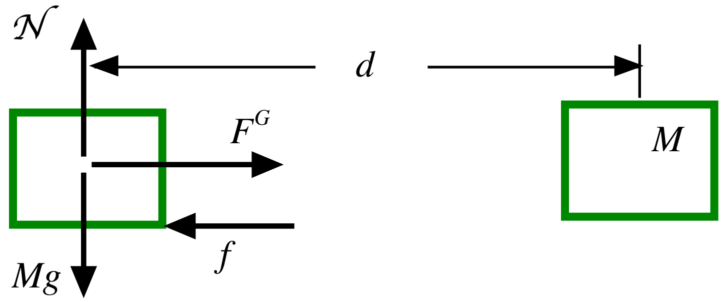
$$f_s \le \mu_s \mathcal{N} \le \mu_s Mg \le (0.600)(100\ \ kg)\left(9.789\ \ m/s^2\right) \le 587\ \ Newtons\ . \tag{4}$$

Initially, the gravitational force is given by

$$F_o^G = \frac{GM^2}{d_o^2} = \frac{\left(6.67\times10^{-11}\,Nm^2/C^2\right)\left(100\ \ kg\right)^2}{\left(1\ \ m\right)^2} = 6.67\times10^{-7}\,N\ . \tag{5}$$

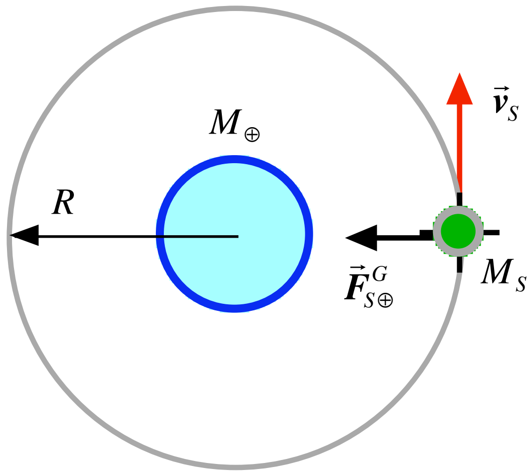
This force is very much smaller than the maximum sustainable static frictional force so the blocks will remain at rest with the static friction equilibrating blocks with a magnitude

$$f = 6.67 \times 10^{-7} N \ . \tag{6}$$



6.) Solution:

a)



The magnitude of the gravitational force exerted on the satellite by the Earth is given by

$$F^G_{S\oplus} = \frac{GM_S M_\oplus}{(3R_\oplus)^2} = \frac{\left(6.67\times10^{-11}\, Nm^2/C^2\right)\left(2.50\times10^5\, kg\right)\left(5.97\times10^{24}\, kg\right)}{\left(3\left(6.378\times10^6\, m\right)\right)^2}$$

$$= 2.72\times10^5\, N \ . \tag{1}$$

b) The magnitude of the acceleration of the satellite is given by

$$a_S = \frac{F^G_{S\oplus}}{M_S} = \frac{GM_\oplus}{(3R_\oplus)^2} = \frac{\left(6.67\times10^{-11}\, Nm^2/C^2\right)\left(5.97\times10^{24}\, kg\right)}{\left(3\left(6.378\times10^6\, m\right)\right)^2} = 1.09\ \frac{m}{s^2} \ . \tag{2}$$

c) As the satellite is moving on a circular orbit, essentially under the influence the Earth's gravitational force only, we can write

$$F^G_{S\oplus} = \frac{GM_S M_\oplus}{(3R_\oplus)^2} = \frac{M_S v_S^2}{3R_\oplus} \ , \tag{3}$$

and

$$v_S = \sqrt{\frac{GM_\oplus}{3R_\oplus}}$$

$$= \sqrt{\frac{\left(6.67\times10^{-11}Nm^2/kg^2\right)\left(5.97\times10^{24}kg\right)}{3\left(6.378\times10^6 m\right)}} = 4,560\ \frac{m}{s} \approx 10,210\ mph\ . \quad (4)$$

d) As the speed is constant for a circular orbit, we have

$$v_S = \frac{2\pi\left(3R_\oplus\right)}{\tau_{orb}} = \sqrt{\frac{GM_\oplus}{3R_\oplus}}\ , \quad (5)$$

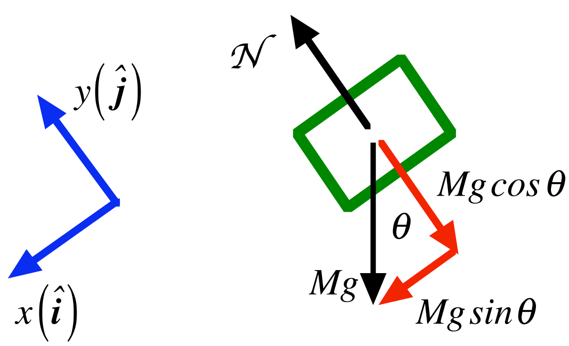
and, therefore,

$$\tau_{orb} = 2\pi\left(3R_\oplus\right)\sqrt{\frac{3R_\oplus}{GM_\oplus}} = 2\pi\sqrt{\frac{\left(3R_\oplus\right)^3}{GM_\oplus}}$$

$$= 2\pi\sqrt{\frac{\left(3\left(6.378\times10^6 m\right)\right)^3}{\left(6.67\times10^{-11}Nm^2/kg^2\right)\left(5.97\times10^{24}kg\right)}} = 2.635\times10^4\ s \equiv 439\ minutes\ . \quad (6)$$

7.) Solution:

a)



b) We can write

$$\mathcal{N} - Mg\cos\theta = Ma_y = 0\ , \quad (1)$$

and

$$\mathcal{N} = Mg\cos\theta\ . \quad (2)$$

c) We have

$$Mg\sin\theta = Ma_x\ , \quad (3)$$

and

$$a_x = g\sin\theta\ . \quad (4)$$

d) As the acceleration is constant, we have

$$v^2 = v_o^2 \pm 2\left|\vec{a}\right|\left|\Delta\vec{r}\right| = 0 + 2\left(g\sin\theta\right)\left(\ell\right), \quad (5)$$

and, therefore,

$$v = \sqrt{2g\ell\sin\theta}\ . \quad (6)$$

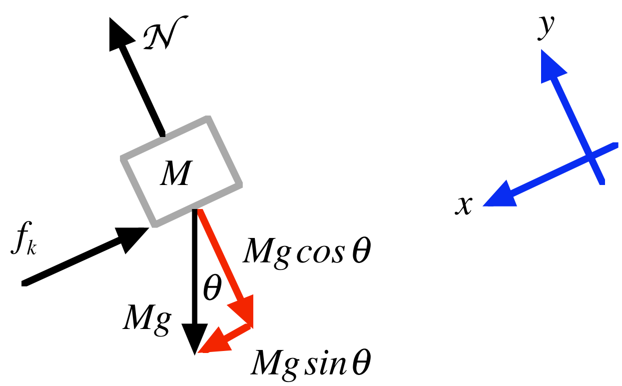
e) (b) $$\mathcal{N} = (2.50\ kg)(9.80\ m/s^2)(\cos 35°) = 20.1\ Newtons\ , \tag{7}$$

(c) $$a_x = (9.80\ m/s^2)(\sin 35°) = 5.62\ m/s^2\ , \tag{8}$$

(d) $$v = \sqrt{2(9.80\ m/s^2)(1.25\ m)(\sin 35°)} = 3.75\ m/s\ . \tag{9}$$

8.) Solution:

a)



b) Looking at the forces directed parallel to y, we find

$$\mathcal{N} - Mg\cos\theta = Ma_y = 0\ , \tag{1}$$

and $$\mathcal{N} = Mg\cos\theta\ . \tag{2}$$

c) Once we know the normal, we know the magnitude of the kinetic friction:

$$f_k = \mu_k \mathcal{N} = \mu_k Mg\cos\theta\ . \tag{3}$$

d) Looking at the forces directed parallel to x, we find

$$Mg\sin\theta - \mu_k Mg\cos\theta = Ma_x\ , \tag{4}$$

and, therefore, $$a_x = (\sin\theta - \mu_k\cos\theta)g\ . \tag{5}$$

e) As the acceleration is constant and the initial speed is zero, we have

$$v = \sqrt{2a\ell} = \sqrt{2(\sin\theta - \mu_k\cos\theta)g\ell}\ . \tag{6}$$

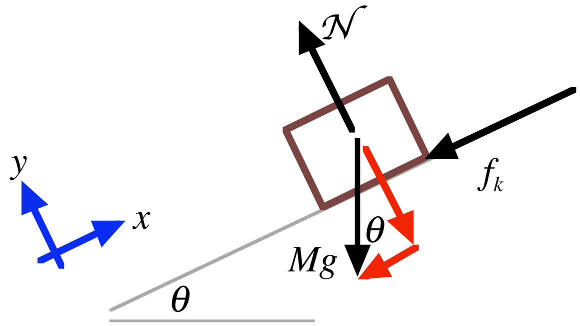
f) (b) $$\mathcal{N} = (2.5\ kg)(9.80\ m/s^2)(\cos 25°) = 22.2\ N\ . \tag{7}$$

(c) $$f_k = (0.3)(2.5\ kg)(9.80\ m/s^2)(\cos 25°) = 6.7\ N\ . \tag{8}$$

(d) $$a_x = [(\sin 25°) - (0.3)(\cos 25°)](9.80\ m/s^2) = 1.48\ m/s^2\ . \tag{9}$$

(e) $$v = \sqrt{2[(\sin 25°) - (0.3)(\cos 25°)](9.80\ m/s^2)(1.25\ m)} = 1.92\ \frac{m}{s}. \tag{10}$$

9.) Solution:



a) Looking at the forces directed parallel to y, we find

$$\mathcal{N} - Mg\cos\theta = Ma_y = 0 \ , \tag{1}$$

and

$$\mathcal{N} = Mg\cos\theta \ . \tag{2}$$

Once we know the normal, we know the kinetic friction:

$$f_k = \mu_k \mathcal{N} = \mu_k Mg\cos\theta \ . \tag{3}$$

Looking at the forces directed parallel to x, we find

$$-Mg\sin\theta - \mu_k Mg\cos\theta = Ma_x \ , \tag{4}$$

and, therefore,

$$\vec{a}_x = -\left(\sin\theta + \mu_k \cos\theta\right) g \ \hat{i} \ . \tag{5}$$

The magnitude of this acceleration is

$$a_x = \left(\sin\theta + \mu_k \cos\theta\right) g$$

$$= \left[\sin 30° + (0.35)\cos 30°\right]\left(9.80 \ m/s^2\right) = 7.87 \ m/s^2 \ . \tag{6}$$

b) As the acceleration is constant and the block is slowing down, we have

$$v^2 = v_o^2 - 2a\ell \ . \tag{7}$$

At the instant the block stops, we have

$$0 = v_o^2 - 2a\ell \ \rightarrow \ \ell = \frac{v_o^2}{2a} = \frac{(12.75 \ m/s)^2}{2\left(7.87 \ m/s^2\right)} = 10.33 \ m \ . \tag{8}$$

c) To find the distance at which the speed is half of its initial value, we use

$$\left(v_o/2\right)^2 = v_o^2 - 2a\ell' \ , \tag{9}$$

$$2a\ell' = v_o^2 - \left(v_o/2\right)^2 = \left(3/4\right)v_o^2 \ , \tag{10}$$

$$\ell' = \frac{(3/4)v_o^2}{2a} = \frac{3}{4}\ell = (0.75)(10.33 \ m) = 7.75 \ m \ . \tag{11}$$

10.) Solution:

a) We write

$$\mathcal{N} - Mg = Ma_y = 0 \ , \tag{1}$$

and

$$\mathcal{N} = Mg \ , \tag{2}$$

from which follows

$$f_k = \mu_k \mathcal{N} = \mu_k Mg \; . \tag{3}$$

So,

$$-\mu_k Mg = Ma_x \; , \tag{4}$$

and
$$\left|a_x\right| = \mu_k g = (0.42)\left(9.80 \;\; m/s^2\right) = 4.12 \;\; m/s^2 . \tag{5}$$

b) We have
$$v_L^2 = 0 = v_o^2 - 2\left|a_x\right|\ell \; , \tag{6}$$

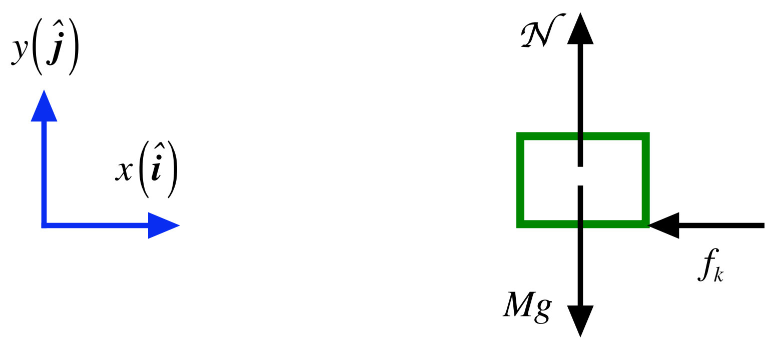
and
$$\ell = \frac{v_o^2}{2\left|a_x\right|} = \frac{\left(15.5 \;\; m/s\right)^2}{2\left(4.12 \;\; m/s^2\right)} = 29.2 \;\; m \; . \tag{7}$$

c) Using (6) again we have

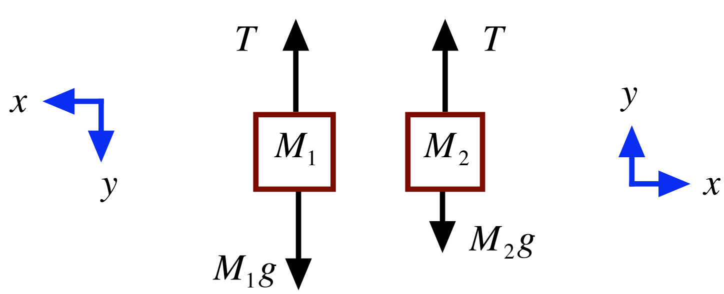
$$v_L^2 = \left(v_o/2\right)^2 = \frac{v_o^2}{4} = v_o^2 - 2\left|a_x\right|\ell' \; , \tag{8}$$

and
$$v_o^2 - \frac{v_o^2}{4} = \frac{3v_o^2}{4} = 2\left|a_x\right|\ell' \, , \tag{9}$$

so that
$$\ell' = \frac{3}{4}\left[\frac{v_o^2}{2\left|a_x\right|}\right] = \frac{3}{4}\ell = (0.75)(29.2 \; m) = 21.9 \;\; m \, . \tag{10}$$



11.) Solution:
a)



b) Using Newton's second law, we write

$$M_1 g - T = M_1 a_1 \; , \tag{1}$$

$$T - M_2 g = M_2 a_2 . \tag{2}$$

Adding equations (1) and (2) gives us

$$M_1 g - T + T - M_2 g = M_1 a_1 + M_2 a_2 , \tag{3}$$

$$M_1 g - M_2 g = M_1 a_1 + M_2 a_2 , \tag{4}$$

$$\left(M_1 - M_2\right) g = M_1 a_1 + M_2 a_2 . \tag{5}$$

At the instant block one has moved downward a distance ℓ, block two has moved exactly the same distance upward. So, the **magnitudes** of their accelerations are equal and

$$a_1 = a_2 = a . \tag{6}$$

Substitution of (6) into (5) yields

$$\left(M_1 - M_2\right) g = \left(M_1 + M_2\right) a , \tag{7}$$

and, therefore,

$$a = \left[\frac{M_1 - M_2}{M_1 + M_2}\right] g . \tag{8}$$

c) We find the tension using equations (2) and (8):

$$T = M_2\left(g + a_2\right) = M_2\left(g + \frac{M_1 - M_2}{M_1 + M_2} g\right) = m_2\left(1 + \frac{M_1 - M_2}{M_1 + M_2}\right) g$$

$$= M_2\left(\frac{M_1 + M_2 + M_1 - M_2}{M_1 + M_2}\right) g = \left[\frac{2 M_1 M_2}{M_1 + M_2}\right] g . \tag{9}$$

d) As the acceleration is constant, we know that

$$v^2 = v_o^2 \pm 2 a \ell , \tag{10}$$

$$v = \sqrt{\left(\frac{M_1 - M_2}{M_1 + M_2}\right) 2 g \ell} . \tag{11}$$

e) (b)

$$a = \left[\frac{3\ kg - 2\ kg}{3\ kg + 2\ kg}\right]\left(9.80\ m/s^2\right) = 1.96\ m/s^2 . \tag{12}$$

(c)

$$T = \left[\frac{2\left(3\ kg\right)\left(2\ kg\right)}{3\ kg + 2\ kg}\right]\left(9.80\ m/s^2\right) = 23.5\ N . \tag{13}$$

(d)

$$v = \sqrt{\left(\frac{3\ kg - 2\ kg}{3\ kg + 2\ kg}\right) 2\left(9.80\ m/s^2\right)\left(0.75\ m\right)} = 1.71\ m/s . \tag{14}$$

12.) Solution:

b)

$$\mathcal{N} - M_1 g = M_1 a_y = 0 , \tag{1}$$

$$\mathcal{N} = M_1 g = \left(8.50\ kg\right)\left(9.80\ m/s^2\right) = 83.3\ N . \tag{2}$$

c)

$$f_k = \mu_k \mathcal{N} = \mu_k M_1 g = \left(0.675\right)\left(8.50\ kg\right)\left(9.80\ m/s^2\right) = 56.2\ N . \tag{3}$$

d) For block one we can write for forces parallel to the x-axis

$$T - \mu_k M_1 g = M_1 a \ , \tag{4}$$

while for block two we have

$$M_2 g - T = M_2 a \ . \tag{5}$$

Adding equations (4) and (5) yields

$$M_2 g - T + T - \mu_k M_1 g = M_1 a + M_2 a \ , \tag{6}$$

and

$$\left(M_2 - \mu_k M_1\right) g = \left(M_1 + M_2\right) a \ . \tag{7}$$

Therefore,

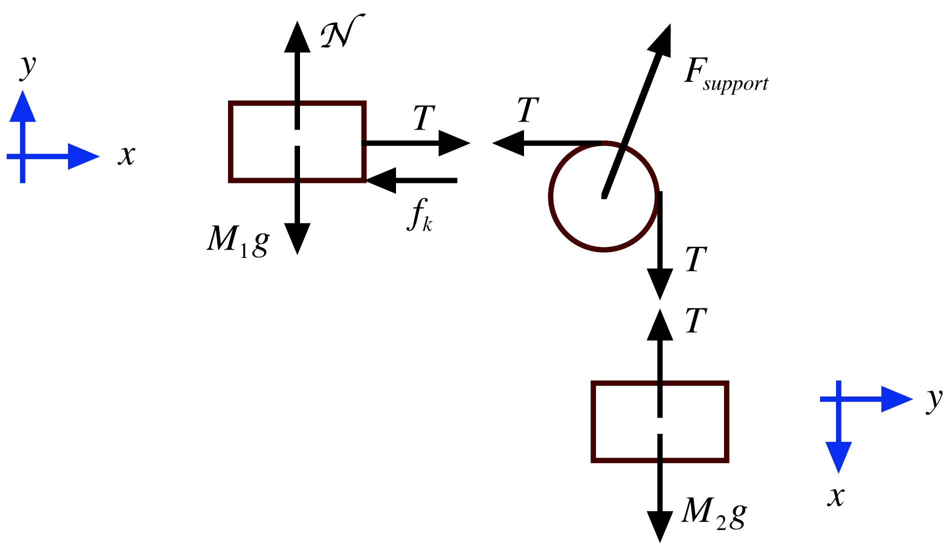
$$a = \left[\frac{M_2 - \mu_k M_1}{M_1 + M_2}\right] g$$

$$= \left[\frac{10.50\ kg - (0.675) 8.50\ kg}{8.50\ kg + 10.50\ kg}\right]\left(9.80\ m/s^2\right) = 2.46 \frac{m}{s^2} \ . \tag{8}$$

e) Using equation (5),

$$T = M_2 (g - a) = (10.50\ kg)\left[\left(9.80\ m/s^2\right) - \left(2.46\ m/s^2\right)\right] = 77.1\ N \ . \tag{9}$$

a)



13.) Solution:

a) Looking at the forces directed parallel to y for block two, we find

$$\mathcal{N} - M_2 g \cos\theta = M_2 a_y = 0 \ , \tag{1}$$

and

$$\mathcal{N} = M_2 g \cos\theta \ . \tag{2}$$

Once we know the normal, we know the kinetic friction:

$$f_k = \mu_k \mathcal{N} = \mu_k M_2 g \cos\theta \ . \tag{3}$$

Looking at the forces directed parallel to x, we find

$$T - M_2 g \sin\theta - \mu_k M_2 g \cos\theta = M_2 a_x \ . \tag{4}$$

Next, we analyze block one. We have

$$M_1 g - T = M_1 a_x \ . \tag{5}$$

Adding equations (4) and (5) we get

$$M_1 g - T + T - M_2 g \sin\theta - \mu_k M_2 g \cos\theta = M_1 a_x + M_2 a_x \ , \tag{6}$$

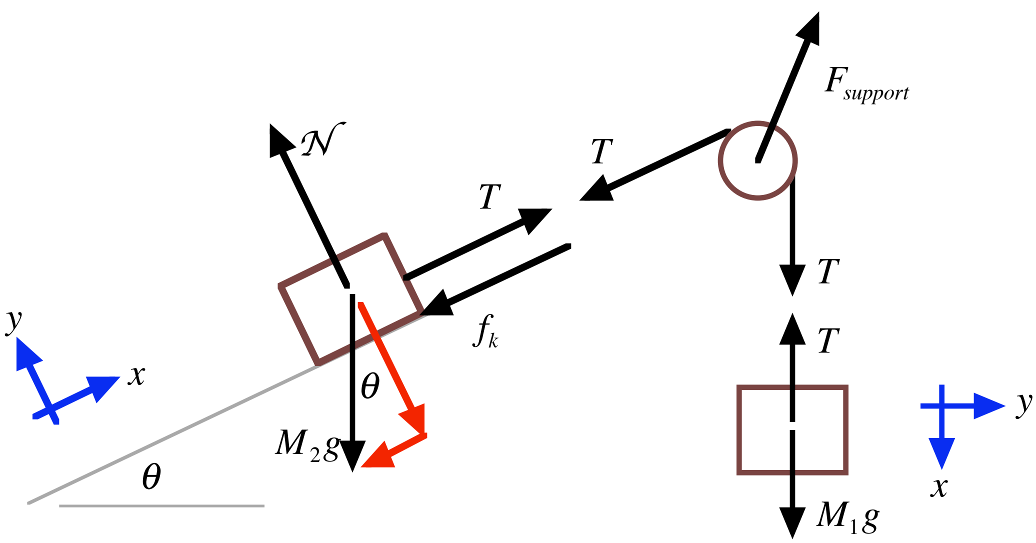
and

$$\left[M_1 - M_2\left(\sin\theta + \mu_k \cos\theta\right)\right]g = \left(M_1 + M_2\right)a_x \ . \tag{7}$$

So,

$$a_x = \left[\frac{M_1 - M_2\left(\sin\theta + \mu_k \cos\theta\right)}{M_1 + M_2}\right]g$$

$$= \left[\frac{(5.5\ kg) - (4.5\ kg)\left(\sin 20^\circ + (0.3)\cos 20^\circ\right)}{(5.5\ kg) + (4.5\ kg)}\right]\left(9.80\ m/s^2\right) = 2.64\frac{m}{s^2} \ . \tag{8}$$



b) Using equation (5), we write

$$T = M_1\left(g - a_x\right) = (5.5\ kg)\left[\left(9.80\ m/s^2\right) - \left(2.64\ m/s^2\right)\right] = 39.4\ N \ . \tag{9}$$

c) As the acceleration is constant and the initial speed is zero, we have

$$v = \sqrt{2a\ell} = \sqrt{2\left(2.64\ m/s^2\right)(0.50\ m)} = 1.62\ m/s \ . \tag{10}$$

14.) Solution: a) As each block moves the same distance in the same time, the magnitude of their acceleration is the same. So, we can write

$$a_1 = a_2 = a \ . \tag{1}$$

Block One:

y direction:

$$\mathcal{N}_1 - M_1 g \cos\theta = M_1 a_{1y} = 0 \ , \tag{2}$$

$$\mathcal{N}_1 = M_1 g \cos\theta \ , \tag{3}$$

$$f_1 = \mu_1 \mathcal{N}_1 = \mu_1 M_1 g \cos\theta \ . \tag{4}$$

x direction:

$$T - \mu_1 M_1 g\cos\theta - M_1 g\sin\theta = M_1 a \ . \qquad (5)$$

In a similar manner for block two, we have

Block Two:

y direction:

$$\mathcal{N}_2 - M_2 g\cos\varphi = M_2 a_{2y} = 0 \ , \qquad (6)$$

$$\mathcal{N}_2 = M_2 g\cos\varphi \ , \qquad (7)$$

$$f_2 = \mu_2 \mathcal{N}_2 = \mu_2 M_2 g\cos\varphi \ . \qquad (8)$$

x direction:

$$M_2 g\sin\varphi - \mu_2 M_2 g\cos\varphi - T = M_2 a \ . \qquad (9)$$

Adding equations (5) and (9) we get

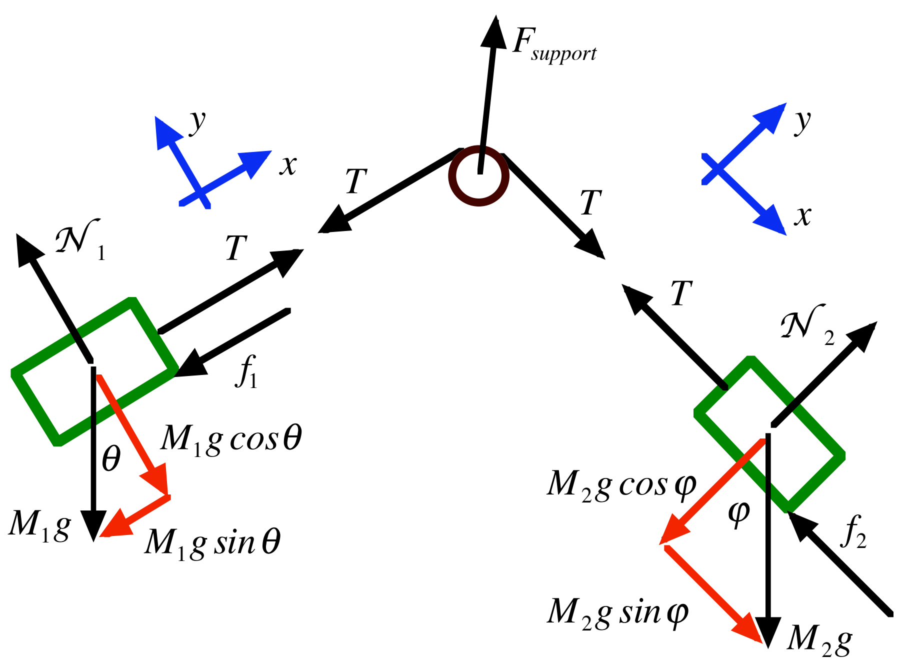
$$\left[M_2\left(\sin\varphi - \mu_2\cos\varphi\right) - M_1\left(\mu_1\cos\theta + \sin\theta\right)\right]g = \left[M_1 + M_2\right]a \ , \qquad (10)$$

and

$$a = \left[\frac{M_2\left(\sin\varphi - \mu_2\cos\varphi\right) - M_1\left(\mu_1\cos\theta + \sin\theta\right)}{M_1 + M_2}\right]g$$

$$= \left[\frac{(8kg)(\sin 45° - .3\cos 45°) - (4kg)(.45\cos 30° + \sin 30°)}{(12kg)}\right]\left(9.80 \ m/s^2\right)$$

$$= 0.328 \ m/s^2 \ . \qquad (11)$$



b) Using equation (5), we have

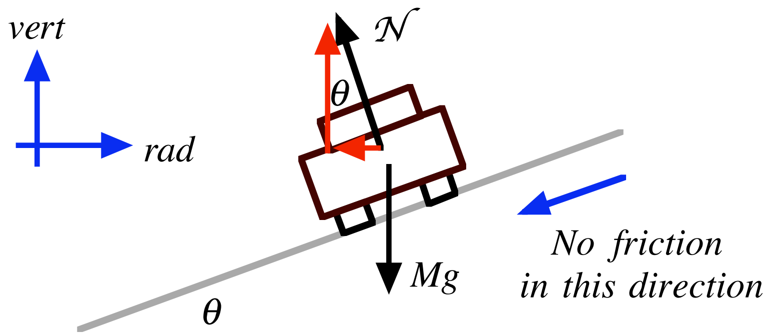
$$T = M_1\left[\left(\mu_1\cos\theta + \sin\theta\right)g + a\right]$$

$$= (4kg)\left[(.45\cos 30° + \sin 30°)\left(9.80 \ m/s^2\right) + \left(0.328 \ m/s^2\right)\right] = 36.4 \ Newtons \ . \qquad (12)$$

c) As the blocks are released from rest, we have

$$v = \sqrt{2|\vec{a}|\ell} = \sqrt{2\left(0.328\ m/s^2\right)\left(.5\ m\right)} = 0.573\ m/s\,. \tag{13}$$

15.) Solution:



Summing the vertical forces, we have

$$\mathcal{N}\cos\theta - Mg = Ma_{vert} = 0\ , \tag{1}$$

and

$$\mathcal{N} = \frac{Mg}{\cos\theta}\ . \tag{2}$$

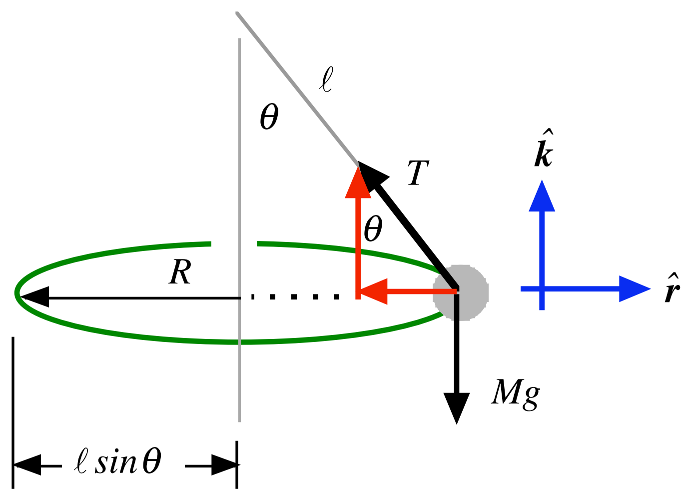
Summing the radial forces, we find

$$-\mathcal{N}\sin\theta = -M\frac{v_{opt}^2}{R} = -\left[\frac{Mg}{\cos\theta}\right]\sin\theta = -Mg\tan\theta\ . \tag{3}$$

Therefore,

$$v_{opt} = \sqrt{Rg\tan\theta}\ . \tag{4}$$

16.) Solution:



a) Adding forces parallel to $\hat{\boldsymbol{k}}$, we have

$$T\cos\theta - Mg = Ma_{vertical} = 0\,, \tag{1}$$

and, therefore,

$$T = \frac{Mg}{cos\theta} = \frac{(2.75\ kg)(9.80\ m/s^2)}{(cos 15°)} = 27.9\ N\ . \quad (2)$$

b) Summing forces in the radial direction, $\hat{\boldsymbol{r}}$, we have

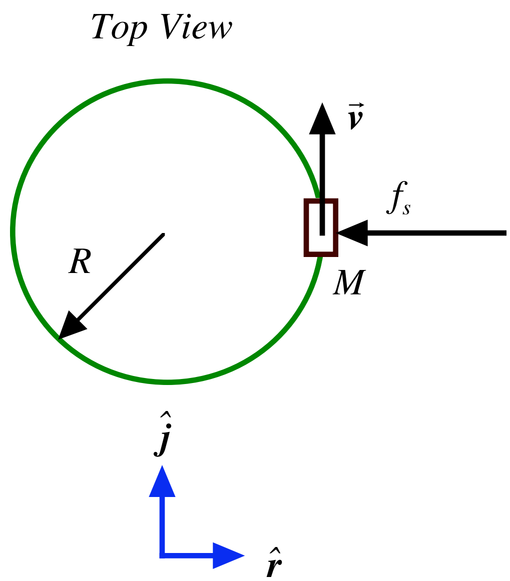
$$-T\ sin\theta = -\frac{Mv^2}{R} = -\frac{Mv^2}{\ell\, sin\theta} = -\left[\frac{Mg}{cos\,\theta}\right] sin\theta = -Mg\, tan\,\theta\ , \quad (3)$$

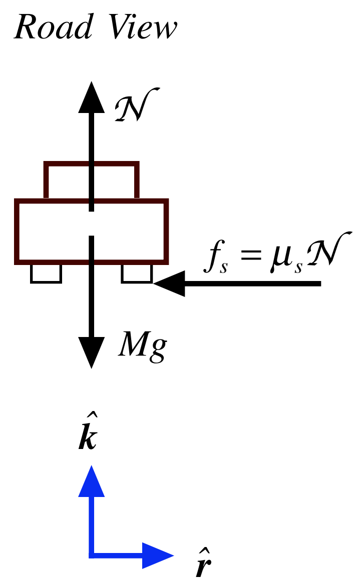
and solving for v gives us

$$v = \sqrt{g\ell\, sin\theta\, tan\theta} = \sqrt{(9.80\ m/s^2)(1.5\ m)(sin 15°)(tan 15°)} = 1.01\ m/s\ . \quad (4)$$

17.) Solution:

a)





b) $$\mathcal{N} - Mg = Ma_y = 0\ , \quad (1)$$

$$\mathcal{N} = Mg\ . \quad (2)$$

c) $$F_{rad} = f_s = \mu_s \mathcal{N} = \mu_s Mg = Mv^2/R = Ma_{rad}\ . \quad (3)$$

d) $$\mu_s Mg = Ma_{rad}\ , \quad (4)$$

$$a_{rad} = \mu_s g\ . \quad (5)$$

e) (b) $$\mathcal{N} = (1.025\times10^3\ kg)(9.80\ m/s^2) = 1.005\times10^4\ N\ . \quad (6)$$

(c) $$F_{rad} = f_s = (0.90)(1.025\times10^3\ kg)(9.80\ m/s^2) = 9.041\times10^3\ N\ . \quad (7)$$

(d) $$a_{rad} = (0.900)(9.80\ m/s^2) = 8.82\ m/s^2\ . \quad (8)$$

f) $$a_{rad} = \frac{v^2}{R} \quad \rightarrow \quad R = \frac{v^2}{a_{rad}} = \frac{v^2}{\mu_s g} = \frac{(20.117\ m/s)^2}{(0.90)(9.8\ m/s^2)} = 45.88\ m\ . \quad (9)$$

18.) Solution:

a) Summing the radially directed forces, we find

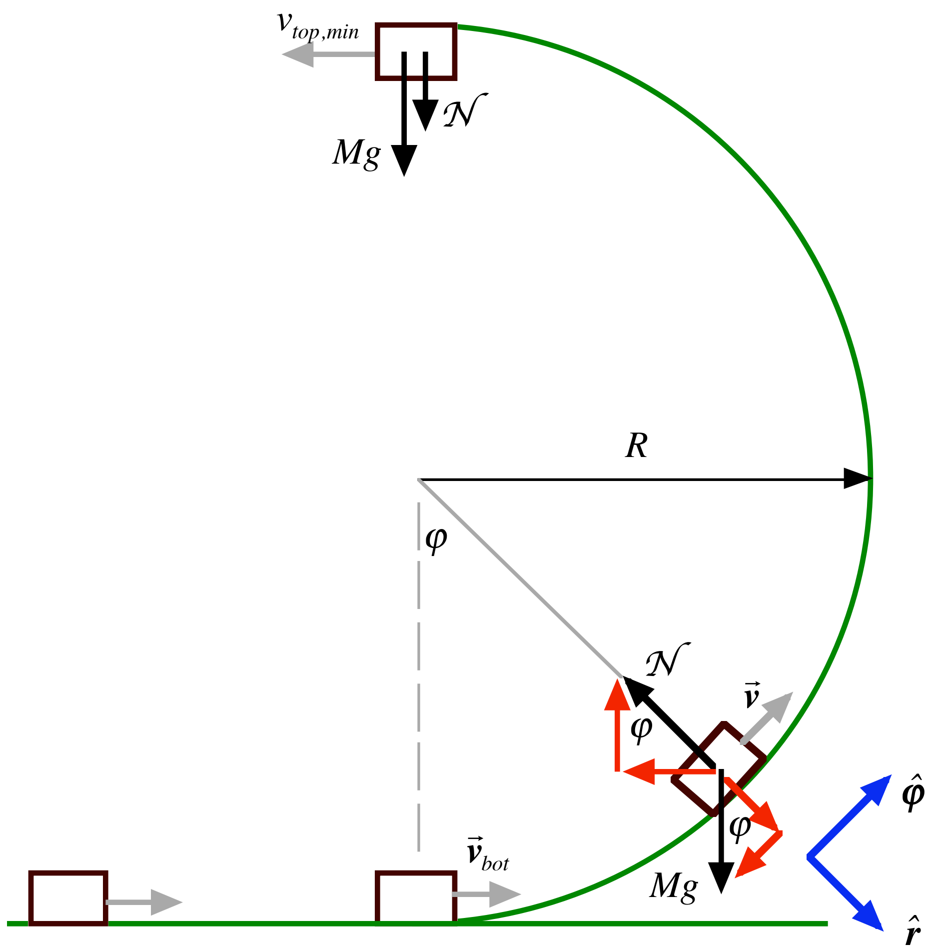
$$-\mathcal{N} + Mg\cos\varphi = -M\frac{v^2}{R} \quad , \tag{1}$$

and

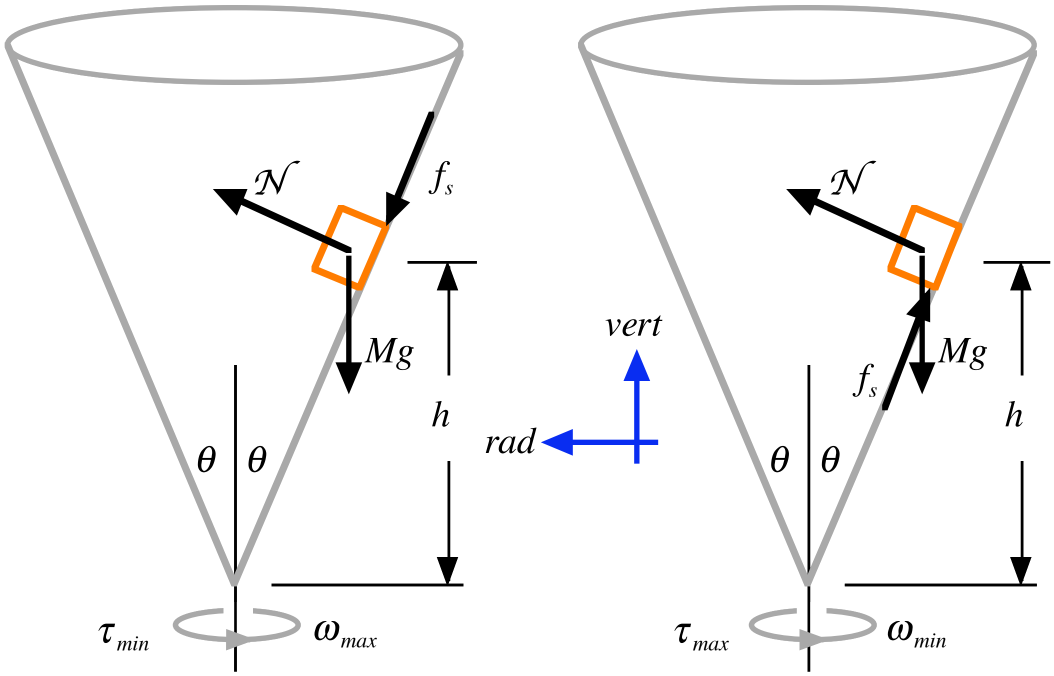
$$v = \sqrt{\frac{\mathcal{N}R}{M} - Rg\cos\varphi} \quad . \tag{2}$$

b) The minimum allowable speed at the top is determined when $\mathcal{N} \to 0$ and $\varphi = 180°$. So, equation (2) becomes

$$v_{top,min} = \sqrt{\frac{(0)R}{M} - Rg(\cos 180°)} = \sqrt{Rg} \quad . \tag{3}$$



19.) Solution:



The minimum period of rotation will require a maximum spin rate. At maximum spin, the block will have impending motion up the surface of the cone. Conversely, the maximum period of rotation will require a minimum spin rate and the block will have impending motion down the surface of the cone. Recall that the physical significance of impending motion is that the static friction has its maximum value and is given by $f_s = \mu_s \mathcal{N}$.

a) Minimum Period:

Summing the vertical forces, we have

$$\mathcal{N}\ sin\theta - \mu_s \mathcal{N}\ cos\theta - Mg = Ma_{vert} = 0 \ ,$$

$$\mathcal{N}\ (sin\theta - \mu_s\ cos\theta) = Mg \, ,$$

and

$$\mathcal{N} = \frac{Mg}{(sin\,\theta - \mu_s\ cos\theta)} \ . \qquad (1)$$

Summing the radial forces, we find

$$-\mathcal{N}\ cos\theta - \mu_s \mathcal{N}\ sin\theta = -MR\omega_{max}^2 = -M\,(h\,tan\theta)\left[\frac{4\pi^2}{\tau_{min}^2}\right] , \qquad (2)$$

$$\mathcal{N}\ (cos\,\theta + \mu_s\ sin\,\theta) = \frac{4\pi^2 M\,(h\,tan\theta)}{\tau_{min}^2} = \left[\frac{cos\,\theta + \mu_s\ sin\,\theta}{sin\,\theta - \mu_s\ cos\,\theta}\right] Mg \, , \qquad (3)$$

where we have used

$$R = h\,tan\theta \ \text{and}\ \omega^2 = (2\pi / \tau_{min})^2 = 4\pi^2 / \tau_{min}^2 \ . \qquad (4)$$

Therefore,

$$\tau_{min} = \sqrt{\frac{4\pi^2 (h\tan\theta)}{g}\left[\frac{\sin\theta - \mu_s \cos\theta}{\cos\theta + \mu_s \sin\theta}\right]} = 2\pi\sqrt{\frac{h\tan\theta}{g}\left[\frac{\tan\theta - \mu_s}{1+\mu_s \tan\theta}\right]} \; . \qquad (5)$$

b) Maximum period:

Summing the vertical forces, we have

$$\mathcal{N} \sin\theta + \mu_s \mathcal{N} \cos\theta - Mg = Ma_{vert} = 0 \; ,$$

$$\mathcal{N} (\sin\theta + \mu_s \cos\theta) = Mg \, ,$$

and

$$\mathcal{N} = \frac{Mg}{(\sin\theta + \mu_s \cos\theta)} \; . \qquad (6)$$

Summing the radial forces, we find

$$-\mathcal{N} \cos\theta + \mu_s \mathcal{N} \sin\theta = -MR\omega_{min}^2 = -M(h\tan\theta)\left[\frac{4\pi^2}{\tau_{max}^2}\right] ,$$

$$\mathcal{N} (\cos\theta - \mu_s \sin\theta) = M(h\tan\theta)\left[\frac{4\pi^2}{\tau_{max}^2}\right] = \left[\frac{\cos\theta - \mu_s \sin\theta}{\sin\theta + \mu_s \cos\theta}\right] Mg \, . \qquad (7)$$

Therefore,

$$\tau_{max} = 2\pi\sqrt{\frac{4\pi^2 h\tan\theta}{g}\left[\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right]} = 2\pi\sqrt{\frac{h\tan\theta}{g}\left[\frac{\tan\theta + \mu_s}{1-\mu_s \tan\theta}\right]} \; . \qquad (6)$$

20.) Solution: Using Newton's universal law of gravitation, we can write

$$F_{S\oplus}^{G} = \frac{GM_S M_\oplus}{R^2} = M_S R\omega^2 = M_S R\left(\frac{2\pi}{\tau}\right)^2 \; . \qquad (1)$$

$$R^3 = (\tau/2\pi)^2 \, GM_\oplus \, , \qquad (2)$$

and

$$R = \left[(\tau/2\pi)^2 \, GM_\oplus\right]^{1/3} \qquad (3)$$

$$= \left[\left(\frac{(24)(3600 \; s)}{2\pi}\right)^2 \left(6.67\times 10^{-11} \, \frac{Nm^2}{kg^2}\right)\left(5.97\times 10^{24} \, kg\right)\right]^{1/3} = 4.223\times 10^7 \, m \, . \qquad (4)$$

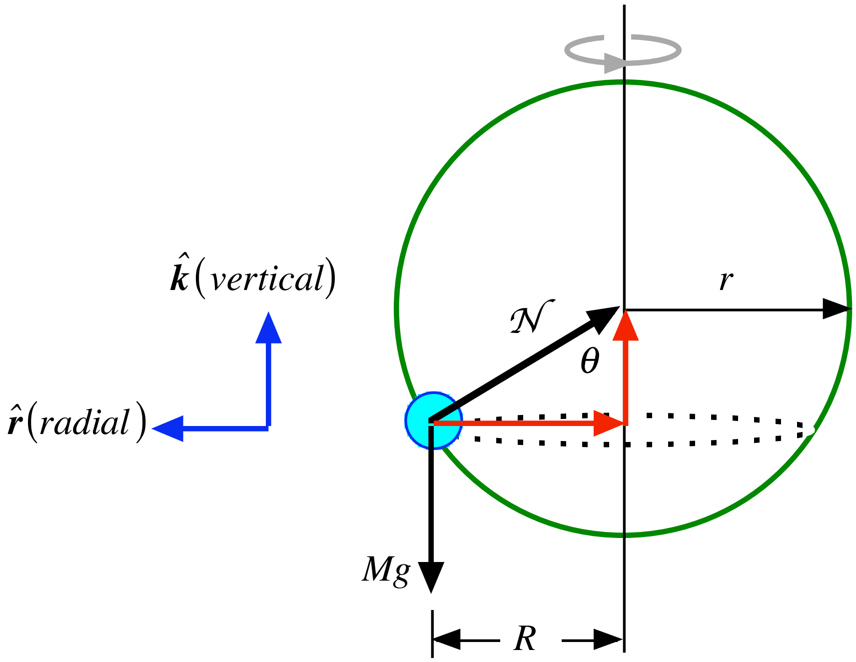
If we now subtract out the radius of the Earth, we will find out how far above the surface of the Earth the satellite "sits." We have

$$d = R - R_\oplus = (4.223\times 10^7 \, m) - (6.378\times 10^6 \, m) = 3.59\times 10^7 \equiv 5.6 R_\oplus \; . \qquad (5)$$

These satellites are "parked" out almost a tenth of the distance to the Moon.

21.) Solution:

a)



b) Applying Newton's second law to the vertically directed forces, we have

$$\mathcal{N}\cos\theta - Mg = Ma_{vertical} = 0 \ , \tag{1}$$

$$\mathcal{N} = \frac{Mg}{\cos\theta} \ . \tag{2}$$

Applying Newton's second law to the radially directed forces, we find

$$-\mathcal{N}\sin\theta = -MR\omega^2 = -M(r\sin\theta)\omega^2 \ , \tag{3}$$

$$-\left[\frac{Mg}{\cos\theta}\right]\sin\theta = -M(r\sin\theta)\omega^2 \ . \tag{4}$$

Solving equation (4) for θ we find

$$\frac{g}{\cos\theta} = r\omega^2 \ , \tag{5}$$

and, therefore,

$$\theta = \cos^{-1}\left[\frac{g}{r\omega^2}\right] = \cos^{-1}\left[\frac{9.80 \ m/s^2}{(0.10 \ m)(3(2\pi \ rad/s))^2}\right] = 74.0° . \tag{6}$$

c) Using equation (3), we write

$$\omega = \sqrt{\frac{g}{r\cos\theta}} = \sqrt{\frac{(9.80 \ m/s^2)}{(0.1 \ m)(\cos 90°)}} = \infty \ ; \ \rightarrow \ \omega = \infty \ . \tag{7}$$

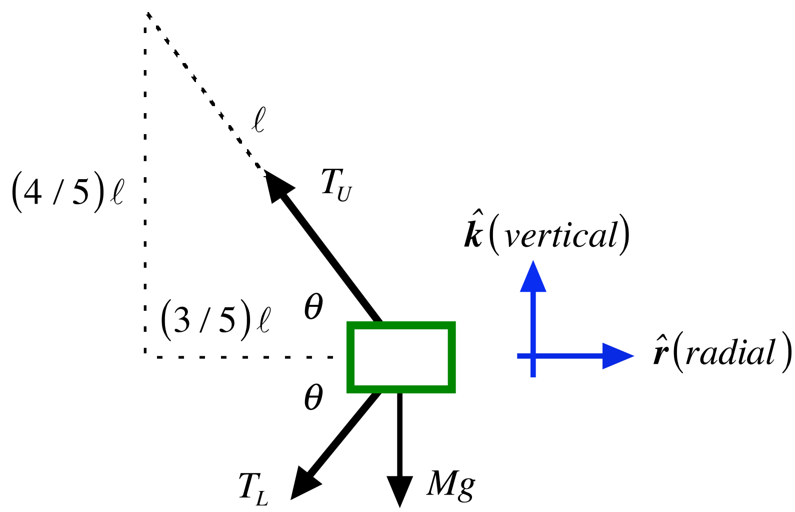
d) For $\theta > 0°$, then we must have $\cos\theta < 1$. So, at $\theta = 0°$,

$$\omega_{min} = \sqrt{\frac{g}{r\cos\theta_{min}}} = \sqrt{\frac{g}{r(\cos 0°)}} = \sqrt{\frac{g}{r}}$$

$$= \sqrt{\frac{(9.80\ m/s^2)}{(0.1\ m)}} = 9.90\ rad/s \equiv \frac{9.90}{2\pi}\frac{rev}{s} = 1.58\ \frac{rev}{s} \equiv 94.5\ rpm\ . \qquad (8)$$

22.) Solution:

a)



c) Applying Newton's second law to the vertically directed forces, we have

$$T_U\ \sin\theta - T_L\ \sin\theta - Mg = Ma_{vertical} = 0\ , \qquad (1)$$

$$T_L = T_U - \frac{Mg}{\sin\theta} = T_U - \frac{5}{4}Mg$$

$$= (90\ N) - \left[(1.25)(4\ kg)(9.80\ m/s^2)\right] = 41.0\ N\ . \qquad (2)$$

b) Applying Newton's second law to the radially directed forces, we find

$$-\ (T_U + T_L)\cos\theta = -MR\omega^2 = -M(\ell\cos\theta)\ \omega^2\ , \qquad (3)$$

$$\omega = \sqrt{\frac{(T_U + T_L)}{M\ell}} = \sqrt{\frac{(90\ N + 41\ N)}{(4\ kg)(1.25\ m)}} = 5.12\ rad/s\ . \qquad (4)$$

As one revolution per minute is related to the angular speed by

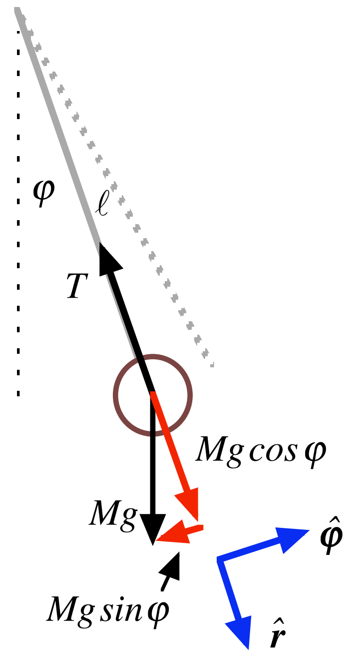
$$1\ rpm \equiv 2\pi\ rad / 60\ s = 0.10472\ rad/s\ , \qquad (6)$$

we can write

$$5.12\ rad/s \equiv 5.12\left(\frac{1}{0.10472}\ rpm\right) = 48.9\ rpm\ . \qquad (7)$$

23.) Solution:

a)



b) For this particular situation, summing the radially directed forces gives us

$$\left[Mg\cos\varphi - T\right]\hat{\boldsymbol{r}} = -\frac{Mv^2}{\ell}\hat{\boldsymbol{r}} = M\vec{\boldsymbol{a}}_{rad} \,, \tag{1}$$

and the tension is given by

$$\vec{\boldsymbol{T}} = -\, M\left[g\cos\varphi + \frac{v^2}{\ell}\right]\hat{\boldsymbol{r}} \,, \tag{2}$$

where the magnitude is

$$T = M\left[g\cos\varphi + \frac{v^2}{\ell}\right], \tag{3}$$

and its direction is

$$\hat{\boldsymbol{T}} = -\hat{\boldsymbol{r}} \,. \tag{4}$$

Further, if we were to sum the tangentially directed forces, we would have

$$M\vec{\boldsymbol{a}}_{tan} = -Mg\sin\varphi\;\hat{\boldsymbol{\varphi}} \,. \tag{5}$$

c) In our coordinate system, the acceleration is given by

$$\vec{\boldsymbol{a}} = \vec{\boldsymbol{a}}_{rad} + \vec{\boldsymbol{a}}_{tan} = -\frac{v^2}{\ell}\hat{\boldsymbol{r}} - g\sin\varphi\;\hat{\boldsymbol{\varphi}} \,, \tag{6}$$

where we have used equations (1) and (5). The magnitude of the acceleration is

$$a = \sqrt{\left(-v^2/\ell\right)^2 + \left(-g\sin\varphi\right)^2} = \sqrt{\left(v^2/\ell\right)^2 + \left(g\sin\varphi\right)^2} \,. \tag{7}$$

The direction of the acceleration is given by

$$\hat{\boldsymbol{a}} = \frac{\vec{\boldsymbol{a}}}{a} = -\frac{\left(v^2 / \ell\right)}{\sqrt{\left(v^2 / \ell\right)^2 + \left(g \sin\varphi\right)^2}}\ \hat{\boldsymbol{r}} - \frac{g \sin\varphi}{\sqrt{\left(v^2 / \ell\right)^2 + \left(g \sin\varphi\right)^2}}\ \hat{\boldsymbol{\varphi}}\ . \tag{8}$$

d) We have at the bottom, $v = v_{bot}$ and $\varphi = 0°$. Substitution of these quantities into equation (6) gives us the acceleration of the sphere at the bottom. We have

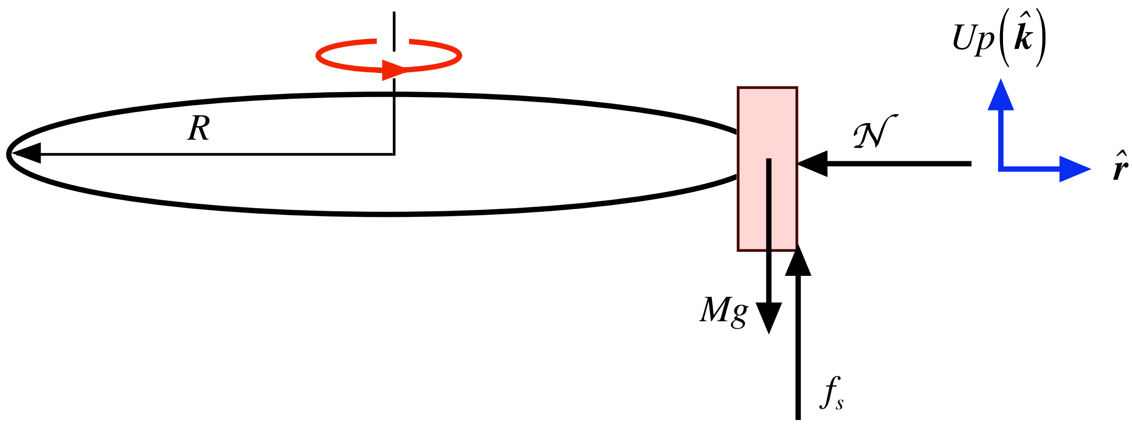
$$\vec{\boldsymbol{a}}_{bot} = -\frac{v_{bot}^2}{\ell}\ \hat{\boldsymbol{r}} - g \sin 0°\ \hat{\boldsymbol{\varphi}} = -\frac{\left(5.486\ m / s\right)^2}{\left(4.572\ m\right)}\ \hat{\boldsymbol{r}} = -\ 6.583\ \frac{m}{s^2}\ \hat{\boldsymbol{r}}\ . \tag{9}$$

e) At the bottom of the path, the magnitude of the tension in the pendulum rope is

$$T = \left(7.256\ kg\right)\left[\left(9.80\ m / s^2\right)\left(\cos 0°\right) + \frac{\left(5.486\ m / s\right)^2}{\left(4.572\ m\right)}\right] = 119\ N\ . \tag{10}$$

24.) Solution:

Free Body Diagram



First, we note that a minimum angular speed would suggest that vertical motion is impending and, therefore, we can write:

$$f_s = \mu_s \mathcal{N} - Mg = Ma_{vert} = 0\ , \tag{1}$$

so

$$\mathcal{N} = \frac{Mg}{\mu_s}\ . \tag{2}$$

Circular motion requires that the magnitude of the net radial force be such that

$$\mathcal{N} = \frac{Mg}{\mu_s} = MR\omega_{min}^2\ , \tag{3}$$

and

$$\omega_{min} = \sqrt{\frac{g}{\mu_s R}}\ . \tag{4}$$

Solutions to Problems for Chapter 11

1.) Solution:

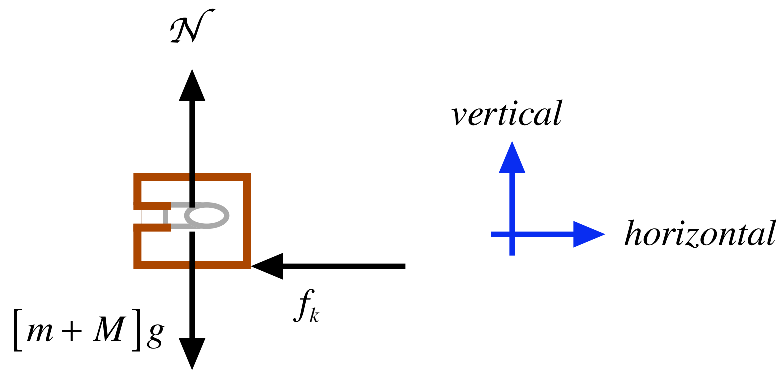
Collision Phase (Conservation of linear momentum.):

$$mv = [m+M]v_{JAC} \ . \qquad (1)$$

$$v_{JAC} = \left[\frac{m}{m+M}\right]v$$

$$= \left[\frac{(0.055\ kg)}{(0.055\ kg)+(5.830\ kg)}\right]\left(380.000\ \frac{m}{s}\right) = 3.55\ \frac{m}{s} \ . \qquad (2)$$

Sliding Phase (Constant acceleration.):



$$\mathcal{N} - [m+M]g = [m+M]a_{vert} = 0 \ . \qquad (3)$$

$$\mathcal{N} = [m+M]g \ . \qquad (4)$$

$$f_k = \mu_k \mathcal{N} = \mu_k [m+M]g \ . \qquad (5)$$

$$-\mu_k [m+M]g = [m+M]a_{horiz} \ , \qquad (6)$$

$$a_{horiz} = -\mu_k g \ . \qquad (7)$$

For the constant acceleration over the sliding interval,

$$v_L^2 = v_o^2 \pm 2a\ell \ , \qquad (8)$$

$$0 = v_{JAC}^2 - 2(\mu_k g)\ell \ , \qquad (9)$$

$$\ell = \frac{(3.55\ m/s)^2}{2(0.82)(9.805\ m/s^2)} = 0.784\ meter \ . \qquad (10)$$

2.) Solution:

Collision Phase (Conservation of linear momentum.):

$$mv_o = Mv_B + m[(1/4)v_o] \ . \qquad (1)$$

$$v_B = \frac{m(v_o - (1/4)v_o)}{M} = \frac{3}{4}\frac{m}{M}v_o = \frac{3}{4}\frac{(0.0150\ kg)}{(2.000\ kg)}(400.00\ m/s) = 2.25\ m/s \ . \qquad (2)$$

3.) Solution:

a) Assume the force is zero at some time t_b. Then we have

$$0 = (400\ N) - (1.333\times 10^5\ N/s)t_b\ , \quad (1)$$

and

$$t_b = \frac{400\ N}{1.333\times 10^5\ N/s} = 3.00\times 10^{-3}\ s\ . \quad (2)$$

b) The average impulse is given by

$$I_{mpulse} = F_{ave}\Delta t = \Delta p\ . \quad (3)$$

The average force is

$$F_{ave} = \frac{1}{2}\left[F(t_o) + F(t_b)\right] = \frac{1}{2}\left[400\ N + 0\ N\right] = 200\ N\ . \quad (4)$$

c) The average impulse is given by

$$I_{mpulse} = (200\ N)(3.00\times 10^{-3} s) = 0.600\ kg\frac{m}{s}\ . \quad (5)$$

d) Using

$$I_{mpulse} = \Delta p = p_f - p_i = m\left[v_f - v_i\right], \quad (6)$$

we have

$$m = \frac{I_{mpulse}}{v_f - v_i} = \frac{I_{mpulse}}{v_f - v_i} = \frac{(0.600\ kg\ m/s)}{(300.00\ m/s) - 0} = 0.0020\ kg\ . \quad (7)$$

Solutions to Problems for Chapter 12

1.) Solution:

a) In *radian* measure, we know

$$\Delta\ell = R\ \Delta\varphi \quad \rightarrow \quad \Delta\varphi = \frac{5.000\ m}{2.000\ m} = 2.500\ radians\ . \tag{1}$$

This measure is equivalent to

$$\Delta\varphi = 2.500\ \ radians \equiv 2.500\left[\frac{180°}{\pi}\right] = 143.24°. \tag{2}$$

b)
$$\Delta\ell = R\ \Delta\varphi = \left(1.500\ m\right)\left(0.600\ rad\right) = 0.900\ m\ . \tag{3}$$

c)
$$R = \frac{\Delta\ell}{\Delta\varphi} = \frac{.450\ m}{\left[42\left(\pi/180\right)rad\right]} = 0.614\ m\ . \tag{4}$$

2.) Solution:

a) Angular speeds must be measured in *radians per second*. So, we have

$$\omega = 3800\left[2\pi\ rad/60\ s\right] = 398\ rad/s\ . \tag{1}$$

b) We can write

$$a_{rad} = R\omega^2 = \left(0.125\ m\right)\left(398\ rad/s\right)^2 = 1.98\times10^4 m/s^2 \equiv 2{,}019\ g's\ . \tag{2}$$

c) As the angular speed is constant, then

$$\omega_{ave} = \Delta\varphi/\Delta t = \omega\ , \tag{3}$$

and
$$\Delta\varphi = \omega\left(\Delta t\right) = \left(398\ rad/s\right)\left(2\ s\right) = 796\ rad \equiv 126.7\ revolutions\ . \tag{4}$$

d)
$$v = R\omega = \left(0.125\ m\right)\left(398\ rad/s\right) = 49.75\ m/s\ . \tag{5}$$

3.) Solution:

As the angular acceleration is constant, we can write

$$\omega = \sqrt{\omega_o^2 + 2\left|\alpha\right|\left|\Delta\varphi\right|}$$

$$= \sqrt{\left(2.2\ rad/s\right)^2 + 2\left|0.2\ rad/s^2\right|\left|3.5\left(2\pi\ rad\right)\right|} = 3.69\ rad/s\ . \tag{1}$$

$$a_{rad} = R\omega^2 = \left(.3302\ m\right)\left(3.69\ rad/s\right)^2 = 4.50\ m/s^2\ . \tag{2}$$

4.) Solution:

a) As the angular acceleration is constant, we can write

$$\omega = \omega_o + \alpha t\ , \tag{1}$$

and, therefore,

$$\alpha = \left(\omega - \omega_o\right)/t\ . \tag{2}$$

Further, we write

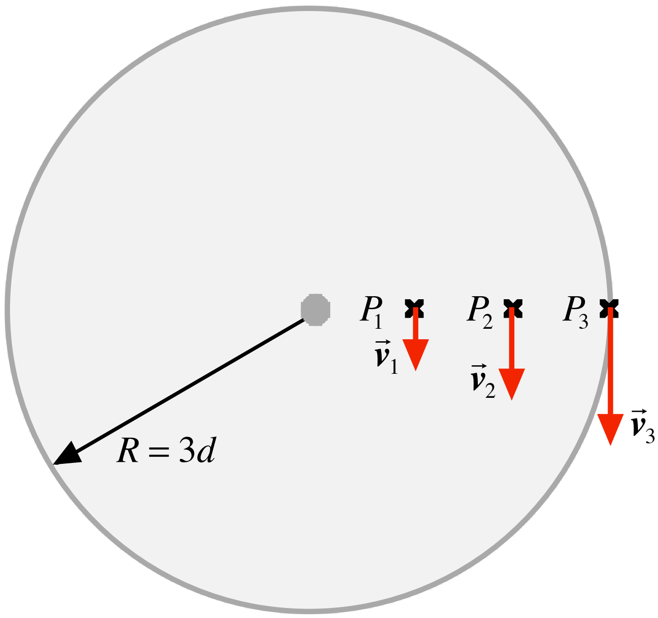
$$\Delta\varphi = \omega_o t + \left(1/2\right)\left[\left(\frac{\omega-\omega_o}{t}\right)\right]t^2 = \omega_o t + \frac{1}{2}\omega t - \frac{1}{2}\omega_o t = \frac{t}{2}\left(\omega+\omega_o\right)\ , \tag{3}$$

and

$$\omega_o = \left[\frac{2(\Delta\varphi)}{t}\right] - \omega = \left[\frac{2(186\ rad)}{(3s)}\right] - \left(108\ \frac{rad}{s}\right) = 16\ \frac{rad}{s}. \tag{4}$$

b)
$$\alpha = \frac{\omega - \omega_o}{t} = \frac{(108\ rad/s) - (16\ rad/s)}{(3\ s)} = 30.67\ \frac{rad}{s^2}\ . \tag{5}$$

5.) Solution:



At point P_1:

a)
$$a_{rad,1} = \left(r_{\perp P_1}\right)\omega^2 = (0.50\ m)(16\ rad/s)^2 = 128\ m/s^2. \tag{1}$$

b)
$$v_1 = \left(r_{\perp P_1}\right)\omega = (0.50\ m)(16\ rad/s) = 8.0\ m/s. \tag{2}$$

At point P_2:

c)
$$a_{rad,2} = \left(r_{\perp P_2}\right)\omega^2 = (1.00\ m)(16\ rad/s)^2 = 256\ m/s^2. \tag{3}$$

d)
$$v_2 = \left(r_{\perp P_2}\right)\omega = (1.00\ m)(16\ rad/s) = 16.0\ m/s. \tag{4}$$

At point P_3:

e)
$$a_{rad,3} = \left(r_{\perp P_3}\right)\omega^2 = (1.50\ m)(16\ rad/s)^2 = 384\ m/s^2. \tag{1}$$

f)
$$v_3 = \left(r_{\perp P3}\right)\omega = (1.50\ m)(16\ rad/s) = 24.0\ m/s. \tag{2}$$

6.) Solution:

a) As the angular acceleration is constant, we have

$$\alpha_{ave} = \frac{\Delta\omega}{\Delta t} = \alpha = \frac{(125\ rad/s) - 0}{(15.25\ s)} = 8.20\ \frac{rad}{s^2}\ . \tag{1}$$

b) $$a_{rad} = R\omega^2 = (0.350\ m)(125\ rad/s)^2 = 5.469\times10^3\ m/s^2 . \quad (2)$$

c) $$v_{tan} = R\omega = (0.350\ m)(125\ rad/s) = 43.75\ m/s\ . \quad (3)$$

7.) Solution:

a) $$\omega_o = (500)(2\pi\ rad/60\ s) = 52.36\ rad/s . \quad (1)$$

$$\omega_L = (3000)(2\pi\ rad/60\ s) = 314.16\ rad/s . \quad (2)$$

b) $$\alpha_{ave} = \Delta\omega/\Delta t = \left[(314.16\ rad/s)-(52.36\ rad/s)\right]/(5\ s) = 52.36\ rad/s^2 . \quad (3)$$

c) $$N_{revolutions} = \frac{\Delta\varphi}{2\pi\ rad} = \frac{(52.36\ rad/s)(5\ s)+(1/2)(52.36\ rad/s^2)(5\ s)^2}{2\pi\ rad}$$

$$= \frac{916.3\ rad}{2\pi\ rad} = 145.8 . \quad (4)$$

d) $$v_{tan} = R\omega = (D/2)\,\omega = \left[\frac{0.295\ m}{2}\right](314.16\ rad/s) = 46.34\ m/s . \quad (5)$$

e) $$a_{rad} = R\omega^2 = \left[\frac{0.295\ m}{2}\right](314.16\ rad/s)^2 = 1.456\times10^4\ m/s^2 . \quad (6)$$

f) $$a_{tan} = R|\alpha| = \left[\frac{0.295\ m}{2}\right](52.36\ rad/s^2) = 7.723\ m/s^2 . \quad (7)$$

Solutions to Problems for Chapter 13

1.) Solution:

In general, the cross product of vectors $\vec{A}$ and $\vec{B}$ is a third vector $\vec{C}$ such that:

$$\begin{aligned}\vec{C} &= \vec{A}\times\vec{B}\\ &=\left[A_x\ \hat{i}+A_y\ \hat{j}+A_z\ \hat{k}\right]\times\left[B_x\ \hat{i}+B_y\ \hat{j}+B_z\ \hat{k}\right]\\ &= A_xB_x\underbrace{\left(\hat{i}\times\hat{i}\right)}_{=0}+A_xB_y\underbrace{\left(\hat{i}\times\hat{j}\right)}_{=\hat{k}}+A_xB_z\underbrace{\left(\hat{i}\times\hat{k}\right)}_{=-\hat{j}}\\ &+A_yB_x\underbrace{\left(\hat{j}\times\hat{i}\right)}_{-\hat{k}}+A_yB_y\underbrace{\left(\hat{j}\times\hat{j}\right)}_{=0}+A_yB_z\underbrace{\left(\hat{j}\times\hat{k}\right)}_{=\hat{i}}\\ &+A_zB_x\underbrace{\left(\hat{k}\times\hat{i}\right)}_{=\hat{j}}+A_zB_y\underbrace{\left(\hat{k}\times\hat{j}\right)}_{=-\hat{i}}+A_zB_z\hat{k}\underbrace{\left(\hat{k}\times\hat{k}\right)}_{=0}\ .\end{aligned} \tag{1}$$

We now collect terms with the same unit vector and we have

$$\begin{aligned}\vec{C} &= \vec{A}\times\vec{B}\\ &=\hat{i}\left(A_yB_z-A_zB_y\right)+\hat{j}\left(A_zB_x-A_xB_z\right)+\hat{k}\left(A_xB_y-A_yB_x\right)\ .\end{aligned} \tag{2}$$

This result is completely general!

So, for the angular momentum, we can write

$$\begin{aligned}\vec{L} &= \vec{r}\times\vec{p}=M\left[\vec{r}\times\vec{v}\right]\\ &= M\left[\hat{i}\left(r_yv_z-r_zv_y\right)+\hat{j}\left(r_zv_x-r_xv_z\right)+\hat{k}\left(r_xv_y-r_yv_x\right)\right]\ .\end{aligned} \tag{3}$$

This process is quite straight forward, but the "bookkeeping" is a royal pain in the proverbial neck! To demonstrate, we have

$$\begin{aligned}\vec{L} &= (1.250\ kg)\left\{\hat{i}\left[(7.00\ m)(-\ 41.17\ m/s)-(-12.00\ m)(-27.43\ m/s)\right]\right.\\ &\quad+\hat{j}\left[(-12.00\ m)(35.75\ m/s)-(4.00\ m)(-\ 41.17\ m/s)\right]\\ &\quad\left.+\hat{k}\left[(4.00\ m)(-27.43\ m/s)-(7.00\ m)(35.75\ m/s)\right]\right\}\\ &= -\ 771.7\ kg\ m/s\ \hat{i}-330.4\ kg\ m/s\ \hat{j}-450.0\ kg\ m/s\ \hat{k}\ .\end{aligned} \tag{4}$$

Note:

$$L=\sqrt{(-771.7)^2+(-330.4)^2+(-450.0)^2}\ kg\ m/s=952.5\ kg\ m/s\ . \tag{5}$$

$$\begin{aligned}\hat{L} &= \frac{\vec{L}}{L}=\frac{-\ 771.7\ kg\ m/s\ \hat{i}-330.4\ kg\ m/s\ \hat{j}-450.0\ kg\ m/s\ \hat{k}}{952.5\ kg\ m/s}\\ &= -\ 0.8102\ \hat{i}-0.3469\ \hat{j}-\ 0.4724\ \hat{k}=cos\,\theta_x\ \hat{i}+cos\theta_y\ \hat{j}+cos\,\theta_z\ \hat{k}\ .\end{aligned} \tag{6}$$

$$\theta_x=cos^{-1}\left[-\ 0.8102\right]=144^\circ\ , \tag{7}$$

$$\theta_y=cos^{-1}\left[-\ 0.3469\right]=110^\circ\ , \tag{8}$$

$$\theta_z = cos^{-1}\left[-\ 0.4724\right] = 118° \ . \tag{9}$$

2.) Solution:

$$L_{\odot,\,spin} = I\omega = \left[\frac{2}{5}M_{\odot}R_{\odot}^2\right]\left[\frac{2\pi}{\tau}\right]$$

$$= \left[\frac{2}{5}\left(1.99\times10^{30}\,kg\right)\left(6.955\times10^{8}\,m\right)^2\right]\left[\frac{2\pi}{(25.05)(24)(3600\ \ s)}\right]$$

$$= 1.12\times10^{42}\,\frac{kg\ \ m^2}{s}\ . \tag{1}$$

3.) Solution:

As the orbit is circular, we know

$$\frac{ke^2}{r^2} = M_{e^-} r\omega^2\ , \tag{1}$$

and, therefore,

$$\omega = \sqrt{\frac{ke^2}{\left(M_{e^-}\right)r^3}}\ . \tag{2}$$

The orbital angular momentum for this point-like object is

$$L_{e^-,orb} = I\omega = \left[\left(M_{e^-}\right)r^2\right]\sqrt{\frac{ke^2}{\left(M_{e^-}\right)r^3}} = \sqrt{ke^2\left(M_{e^-}\right)r}$$

$$= \sqrt{\left(8.99\times10^{9}\,Nm^2\ /\ C^2\right)\left(1.602\times10^{-19}\,C\right)^2\left(9.11\times10^{-31}\,kg\right)\left(5.3\times10^{-11}\,m\right)}$$

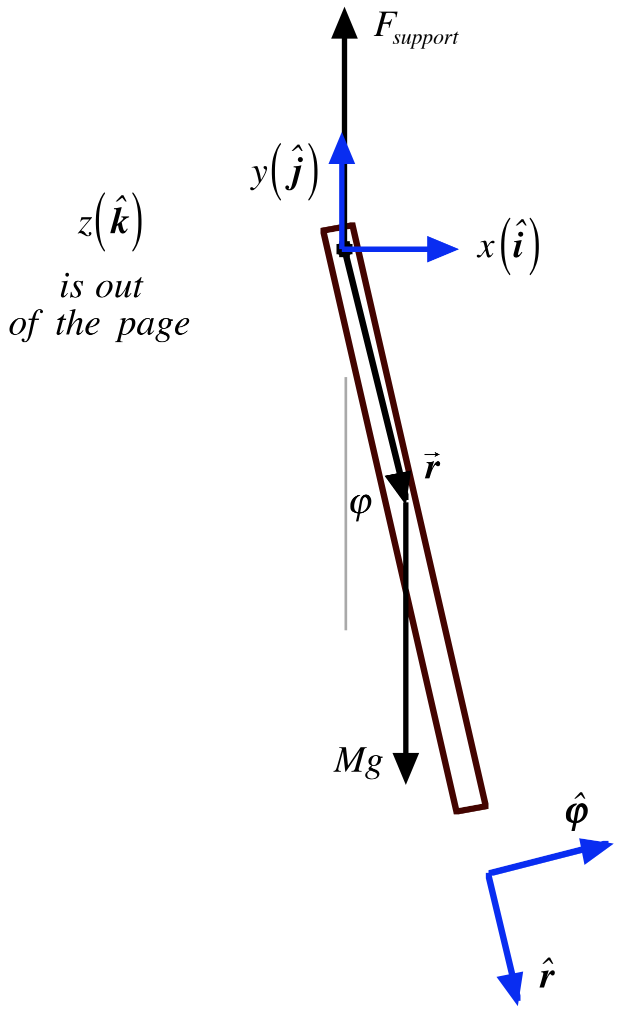
$$= 1.06\times10^{-34}\,\frac{kg\ \ m^2}{s}\ . \tag{3}$$

(This result is slightly different from the measured value because our calculation did not take into consideration that the electron is so small that its behavior is better described by quantum mechanics than classical mechanics.)

Solutions to Problems for Chapter 14

1.) Solution:

a)



b) We know

$$\vec{\boldsymbol{\Gamma}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}$$

$$= \left[(\ell/2)\hat{\boldsymbol{r}}\right] \times \left[-Mg\ \hat{\boldsymbol{j}}\right] = (\ell/2)\,Mg\left[\hat{\boldsymbol{r}} \times -\hat{\boldsymbol{j}}\right] = -(\ell/2)\,Mg\sin\varphi\ \hat{\boldsymbol{k}}\ . \tag{1}$$

So,

$$\Gamma = I\alpha = \left[(1/3)\,M\ell^2\right]\alpha\ . \tag{3}$$

So, we have

$$\alpha = \frac{\Gamma}{I} = \frac{\left(-(\ell/2)Mg\sin\varphi\right)}{(1/3)M\ell^2} = -\frac{3}{2}\frac{g\sin\varphi}{\ell}\ . \tag{4}$$

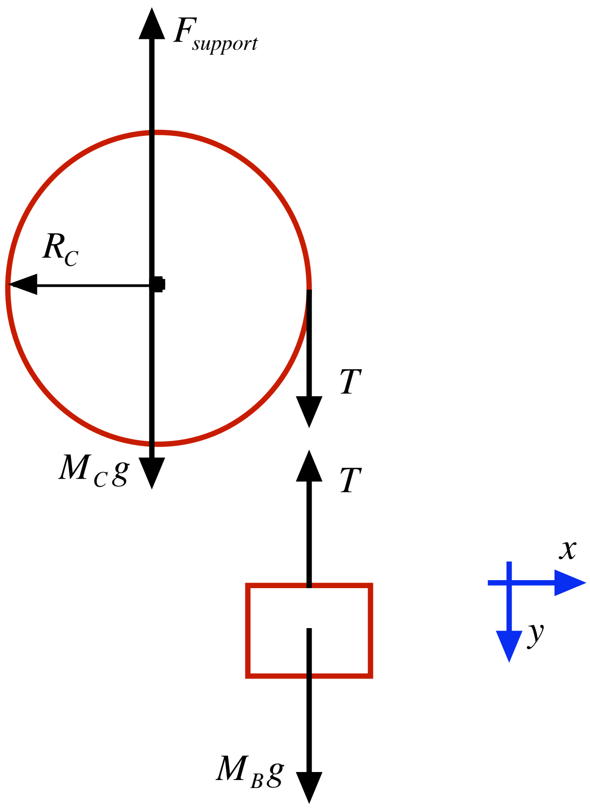
c) Initially, $\varphi = \varphi_o = 25^\circ$ and, therefore,

$$|\alpha_o| = \left| -\frac{3}{2}\frac{g\, sin\varphi_o}{\ell} \right| = \frac{(1.5)(9.805\ \ m/s^2)(sin\,25°)}{(2.250\ \ m)} = 2.76\ rad/s\,. \qquad (5)$$

d) As the rod passes through the vertical, $\varphi = 0°$ and the angular acceleration is zero.

2.) Solution:

a)



b) We can write, using Newton's second law,

$$M_B g - T = M_B a_y\,. \qquad (1)$$

For the cylinder,

$$R_C T = I\alpha = \left[\frac{1}{2}M_C R_C^2\right]\left[\frac{a_y}{R_C}\right], \qquad (2)$$

and, therefore,

$$T = \frac{1}{2}M_C a_y\ . \qquad (3)$$

Substitution of equation (3) into (1) yields

$$M_b g - \frac{1}{2}M_c a_y = M_b a_y\ , \qquad (4)$$

and

$$a_y = \left[\frac{M_B}{(1/2)M_C + M_B} \right] g$$

$$= \left[\frac{(5\ kg)}{(1/2)(9.5\ kg) + (5\ kg)} \right] (9.805\ m/s^2) = 5.028\ \frac{m}{s^2}\ . \qquad (5)$$

c) $$\alpha = \frac{a_y}{R} = \frac{5.028\ m/s^2}{0.150\ m} = 33.5\ rad/s^2\ . \qquad (6)$$

d) $$v = \sqrt{2 a_y \ell} = \sqrt{2(5.028\ m/s^2)(3\ m)} = 5.49\ m/s\ . \qquad (7)$$

3.) Solution:

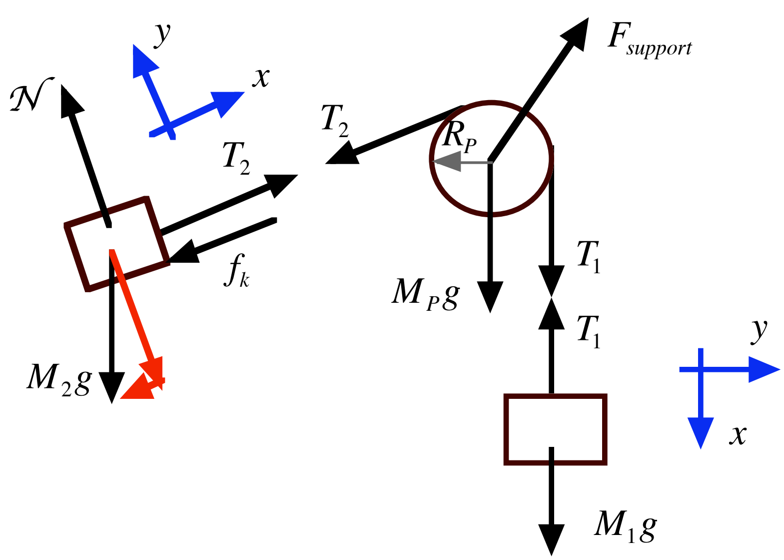
a) $$\Gamma = rF = (0.06\ m)(76.08\ N) = 4.56\ mN\ . \qquad (1)$$

b) $$\alpha = \frac{\Gamma}{I} = \frac{4.56\ mN}{(1/2)(4\ kg)(0.18\ m)^2} = 70.4\ rad/s^2\ . \qquad (2)$$

c) $$\omega = \sqrt{2\alpha(\Delta\varphi)} = \sqrt{2(70.4\ rad/s^2)(147\ rad)} = 144\ rad/s\ . \qquad (3)$$

4.) Solution:

a)



b) Each block moves the same distance in the same amount of time. So, each block has the same magnitude of acceleration that we will signify with a. We can write for block one:

$$M_1 g - T_1 = M_1 a\ , \qquad (1)$$

and

$$T_1 = M_1 g - M_1 a\ . \qquad (2)$$

For block two:

y: $$\mathcal{N} - M_2 g \cos\theta = M_2 a_y = 0\ , \qquad (3)$$

$$\mathcal{N} = M_2 g\, cos\theta \ , \tag{4}$$

and

$$f_k = \mu_k \mathcal{N} = \mu_k M_2 g\, cos\,\theta \ . \tag{5}$$

x:

$$T_2 - M_2 g\, sin\theta - \mu_k M_2 g\, cos\theta = M_2 a \ , \tag{6}$$

and

$$T_2 = M_2 g\, sin\theta + \mu_k M_2 g\, cos\theta + M_2 a \ . \tag{7}$$

Analysis of the rotating pulley reveals

$$RT_1 - RT_2 = I\,|\alpha| = \left[\frac{1}{2} M_P R^2\right]\left[\frac{a}{R}\right] , \tag{8}$$

and, therefore,

$$T_1 - T_2 = \frac{1}{2} M_P a \ . \tag{9}$$

Substitution of equations (2) and (7) into (9) gives us

$$\left[M_1 g - M_1 a\right] - \left[M_2 g\, sin\theta + \mu_k M_2 g\, cos\theta + M_2 a\right] = \frac{1}{2} M_P a \ . \tag{10}$$

Collecting terms,

$$\left[M_1 - M_2\left(sin\theta + \mu_k\, cos\theta\right)\right] g = \left[M_1 + M_2 + \left(1/2\right) M_P\right] a \ . \tag{11}$$

Therefore,

$$a = \left[\frac{M_1 - M_2\left(sin\,\theta + \mu_k\, cos\,\theta\right)}{M_1 + M_2 + \left(1/2\right) M_P}\right] g$$

$$a = \left[\frac{\left(8\ kg\right) - \left(4\ kg\right)\left(sin\,25° + \left(.25\right) cos\,25°\right)}{\left(8\ kg\right) + \left(4\ kg\right) + \left(1/2\right)\left(2\ kg\right)}\right]\left(9.805\ m/s^2\right) = 4.075 \frac{m}{s^2} \ . \tag{12}$$

c) As the system was released from rest and is subject to a constant acceleration, we can write

$$v = \sqrt{2a\ell'} = \sqrt{2\left(4.075\ m/s^2\right)\left(5.75\ m\right)} = 6.85\ m/s \ . \tag{13}$$

5.) Challenge Problem:
Solution:

b) The blocks do not accelerate at the same magnitude because they do not move through equal distances in equal time intervals. The diagram below illustrates this. The red points indicate how far block two will move in time interval Δt and the blue points indicate how far block one will move in this time interval. We have

$$\Delta\ell' = r\left(\Delta\varphi\right) , \tag{1}$$

and

$$\Delta\ell = R\left(\Delta\varphi\right) \ . \tag{2}$$

Using equations (1) and (2) we can write

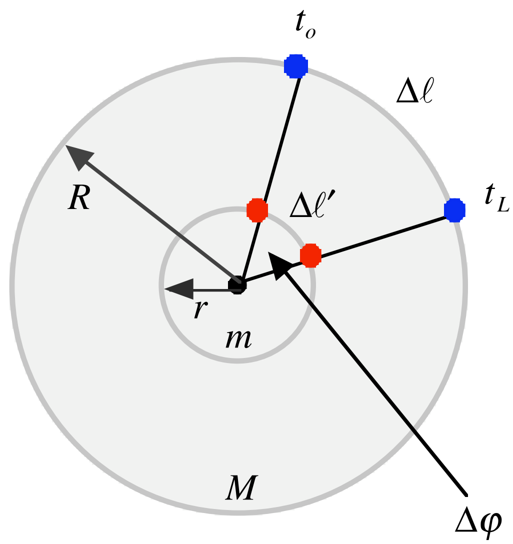
$$\left(\Delta\varphi\right) = \frac{\Delta\ell'}{r} = \frac{\Delta\ell}{R} \ . \tag{3}$$

Since $R = 3r$, we can use equation (3) to write

$$\Delta\ell = \left[\frac{R}{r}\right]\Delta\ell' = 3\ \Delta\ell' . \quad (4)$$

So, block one moves three times farther that block two in the same time interval. Therefore,

$$a_1 = 3a_2 = a . \quad (5)$$



Applying Newton's second law to block one, we find

$$M_1 g - T_1 = M_1 a_1 , \quad (6)$$

and

$$T_1 = M_1 g - M_1 a_1 \quad (7)$$

Block two gives us

$$T_2 - M_2 g = M_2 a_2 = M_2\left[(1/3)a_1\right] = (1/3)M_2 a_1 , \quad (8)$$

$$T_2 = M_2 g + (1/3)M_2 a_1 . \quad (9)$$

An analysis of the rotational dynamics of the disks gives us

$$RT_1 - rT_2 = I\,|\alpha| = \left[(1/2)mr^2 + (1/2)MR^2\right]\left[\frac{a_1}{R}\right]. \quad (10)$$

Using the values given, we can rewrite equation (10) as

$$RT_1 - (1/3)RT_2 = \left[(1/2)(1/9)M\left((1/3)R\right)^2 + (1/2)MR^2\right]\left[\frac{a_1}{R}\right], \quad (11)$$

and, to help clarify,

$$R\left[T_1 - \frac{1}{3}T_2\right] = \left[\left(\frac{1}{162} + \frac{1}{2}\right)MR^2\right]\left[\frac{a_1}{R}\right], \quad (12)$$

$$T_1 - (1/3)T_2 = (41/81)Ma_1 . \quad (13)$$

Substitution of equations (7) and (9) into (13) gives us

$$\left[M_1 g - M_1 a_1\right] - (1/3)\left[M_2 g + (1/3)M_2 a_1\right] = (41/81)Ma_1 . \quad (14)$$

Collecting and rearranging terms,

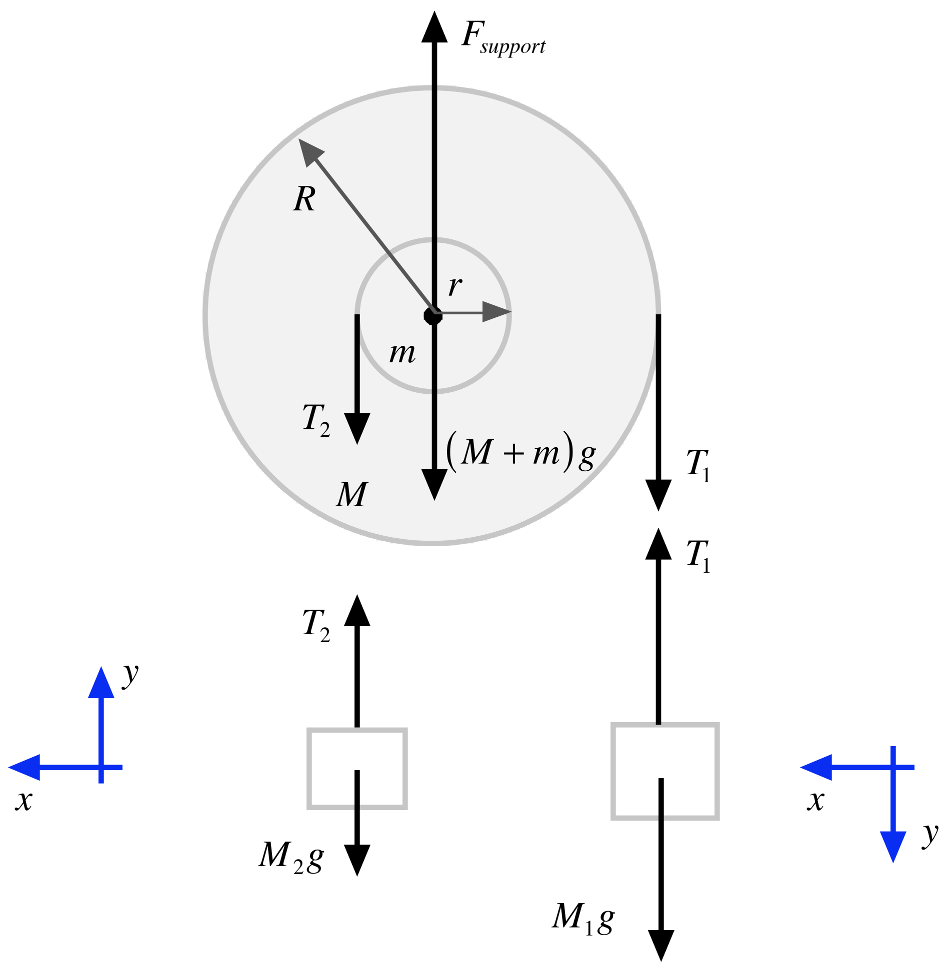
$$\left[M_1 - (1/3)M_2\right]g = \left[M_1 + (1/9)M_2 + (41/81)M\right]a_1 , \quad (15)$$

$$a_1 = \left[\frac{M_1 - (1/3)M_2}{M_1 + (1/9)M_2 + (41/81)M} \right] g$$

$$a_1 = \left[\frac{(18\ kg) - (1/3)(6\ kg)}{(18\ kg) + (1/9)(6\ kg) + (41/81)(18\ kg)} \right] (9.805\ m/s^2) = 5.648\ \frac{m}{s^2}. \quad (16)$$

and
$$a_2 = (1/3)a_1 = (1/3)(5.648\ m/s^2) = 1.883\ m/s^2. \quad (17)$$

a)



c)

$$v_1 = \sqrt{2a_1\ell} = \sqrt{2(5.648\ m/s^2)(0.75\ m)} = 2.911\ m/s. \quad (18)$$

$$v_2 = \sqrt{2a_2\ell_2} = \sqrt{2(a_1/3)(\ell_1/3)} = \sqrt{2a_1\ell_1/9} = v_1/3 = 0.970\ m/s. \quad (19)$$

6.) Solution:

b) As each block moves the same distance in a given time interval, the magnitudes of the accelerations are equal. I will signify this acceleration with a. For block one, we write:

y: $$\mathcal{N} - M_1 g = M_1 a_y = 0 \ , \quad (1)$$

$$\mathcal{N} = M_1 g \ , \quad (2)$$

$$f_k = \mu_k \mathcal{N} = \mu_k M_1 g \ . \quad (3)$$

x: $$T_1 - \mu_k M_1 g = M_1 a \ , \quad (4)$$

$$T_1 = \mu_k M_1 g + M_1 a \ . \quad (5)$$

For block two:

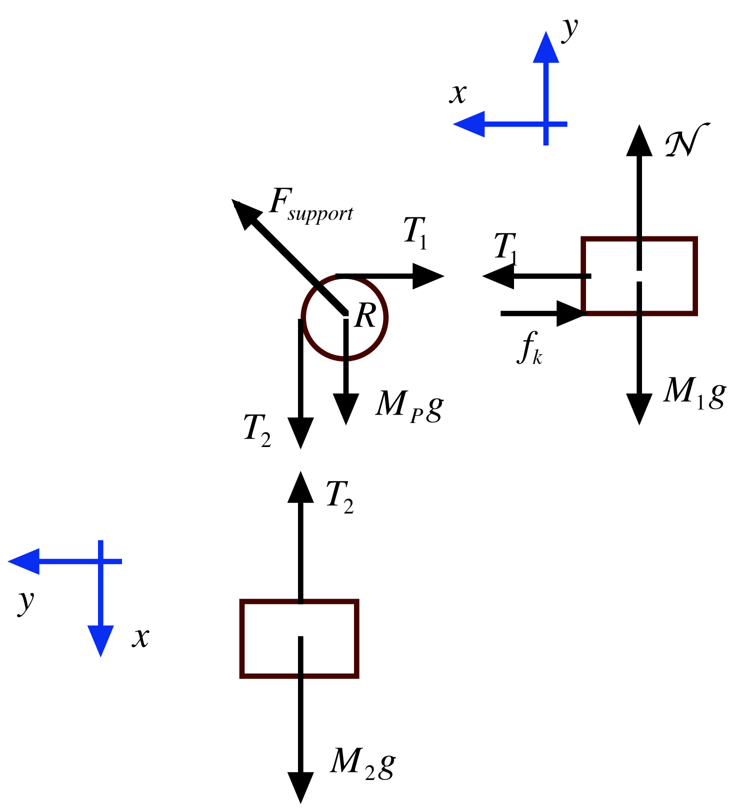
x: $$M_2 g - T_2 = M_2 a \ , \quad (6)$$

$$T_2 = M_2 g - M_2 a \ . \quad (7)$$

Next, we analyze the rotational dynamics of the pulley. We sum the torques and find

$$RT_2 - RT_1 = I\left|\alpha\right| = \left[(1/2)M_p R^2\right]\left[a/R\right] \ . \quad (8)$$

a)



b) continued:

Substitution of the equations for the tensions leads us to

$$M_2 g - M_2 a - \mu_k M_1 g - M_1 a = (1/2) M_p a . \qquad (9)$$

Collecting and rearranging terms, we find

$$[M_2 - \mu_k M_1] g = [M_1 + M_2 + (1/2) M_p] a , \qquad (10)$$

and

$$a = \left[\frac{M_2 - \mu_k M_1}{M_1 + M_2 + (1/2) M_p} \right] g$$

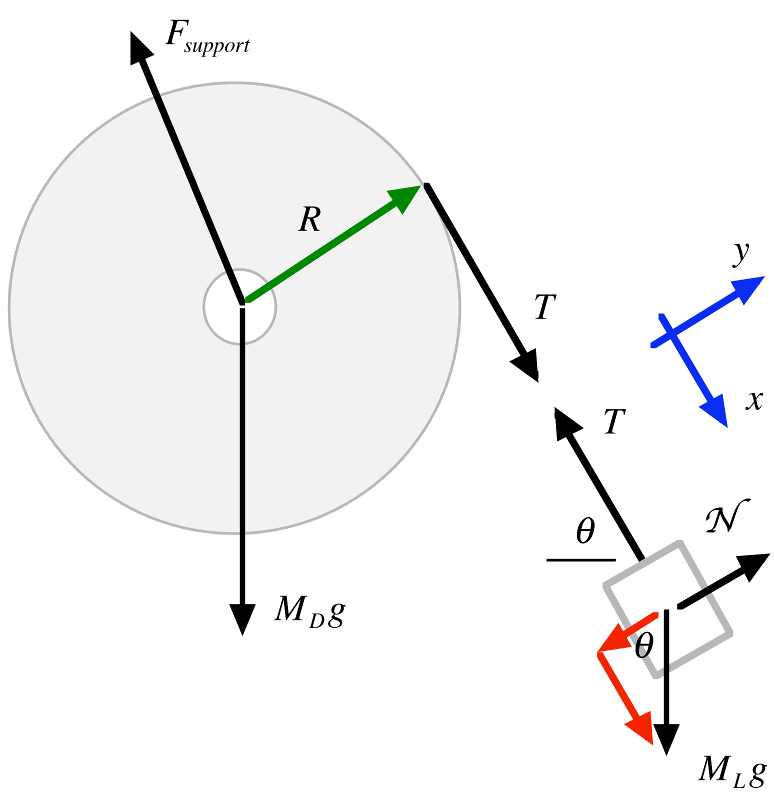
$$= \left[\frac{(9\ kg) - (.352)(6\ kg)}{(6\ kg) + (9\ kg) + (1/2)(2\ kg)} \right] (9.805\ m/s^2) = 4.221 \frac{m}{s^2} . \qquad (11)$$

c) As the blocks are released from rest and subject to a constant acceleration, we have

$$v = \sqrt{2(4.221\ m/s^2)(0.75\ m)} = 2.52\ m/s\ . \qquad (12)$$

7.) Solution:

a)



For the block, we can write

$$M_L g \sin\theta - T = M_L a_x \ . \tag{1}$$

An analysis of the rotational dynamics of the disk gives us

$$RT = I|\alpha| = \left[(1/2)M_D R^2\right]\left[a_x / R\right] \ , \tag{2}$$

and

$$T = (1/2)M_D a_x . \tag{3}$$

Equations (1) and (3) imply

$$M_L g \sin\theta - (1/2)M_D a_x = M_L a_x \ , \tag{4}$$

and

$$M_L g \sin\theta = \left(M_L + (1/2)M_D\right)a_x \ . \tag{5}$$

Therefore,

$$a_x = \left[\frac{M_L}{M_L + (1/2)M_D}\right] g \sin\theta$$

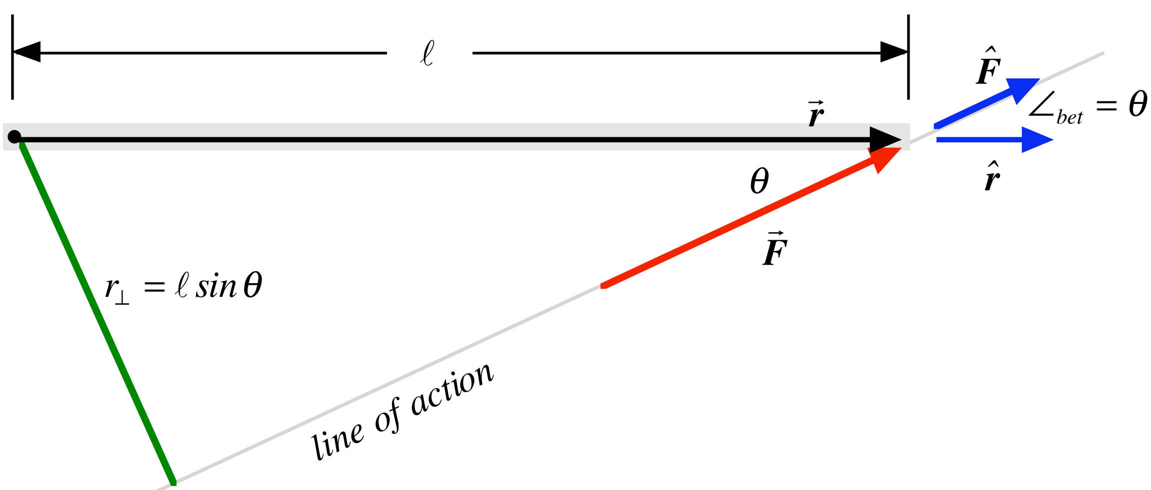
$$= \left[\frac{(125\ kg)}{(125\ kg)+(1/2)(75\ kg)}\right](9.805\ m/s^2)(\sin 60°) = 6.532\ m/s^2 \ . \tag{6}$$

b) $$|\alpha| = |\vec{a}_x|/R = (6.532\ m/s^2)/(0.45\ m) = 14.5\ rad/s^2 \ . \tag{7}$$

c) For a physical thing initially at rest and subject to a constant acceleration, the later speed is

$$v = \sqrt{2a(\Delta r)} = \sqrt{2(6.532\ m/s^2)(1.250\ m)} = 4.041\ m/s \ . \tag{8}$$

8.) Solution:



a) We can write

$$\vec{\Gamma} = \vec{r} \times \vec{F} = [\ell\ \hat{r}] \times [F\ \hat{F}] = \ell F[\hat{r} \times \hat{F}] = (\ell \sin\theta) F\ \hat{\Gamma} \ , \tag{1}$$

and,

$$\Gamma = (\ell \sin\theta)F = [(2\ m)(\sin 25°)](21.5\ N) = 18.2\ mN . \tag{2}$$

b)

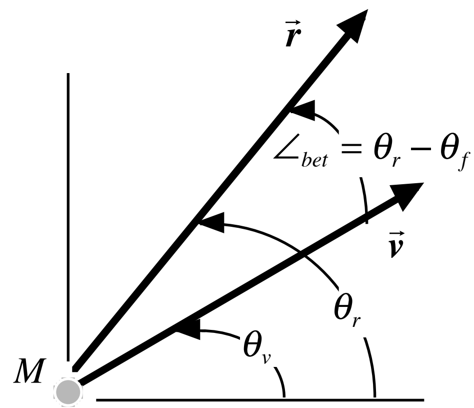
$$\Gamma = I\left|\alpha\right| , \tag{3}$$

and

$$\left|\alpha\right| = \frac{\Gamma}{I} = \frac{(\ell \sin\theta) F}{(1/3) M\ell^2} = \frac{3F \sin\theta}{M\ell}$$

$$= \frac{3(21.5\ N)(\sin 25^\circ)}{(12\ kg)(2\ m)} = 1.136 \frac{rad}{s^2} . \tag{4}$$

9.) Solution:



a)
$$\vec{p} = M\vec{v} = (3.234\ kg)\left\{(26.500\ m/s)\left[\cos 30^\circ\ \hat{i} + \cos 60^\circ\ \hat{j}\right]\right\}$$

$$= 74.22\ \frac{kg\ m}{s}\ \hat{i} + 42.85\ \frac{kg\ m}{s}\ \hat{j} . \tag{1}$$

Note:

$$p = \sqrt{(74.22)^2 + (42.85)^2}\ kg(m/s) = 85.7\ kg(m/s) . \tag{2}$$

b)
$$\vec{L} = \vec{r} \times \vec{p} = M\left[\vec{r} \times \vec{v}\right] = r(Mv)\left[\hat{r} \times \hat{v}\right] = -\ rp(\sin\angle_{bet})\ \hat{k} . \tag{3}$$

Note:

$$\theta_r = \tan^{-1}\left[5/4\right] = 51.340^\circ , \tag{4}$$

while

$$\theta_v = 30^\circ . \tag{5}$$

So, we have

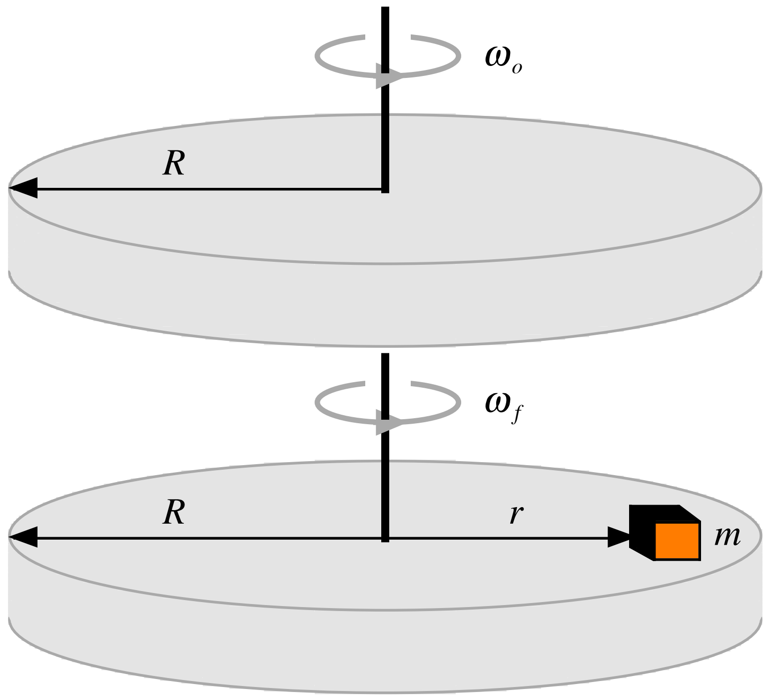
$$\angle_{bet} = \theta_r - \theta_v = 21.340^\circ . \tag{6}$$

Finally,

$$\vec{L} = -\left(\sqrt{(4\ m)^2 + (5\ m)^2}\right)\left[85.7\ kg(m/s)\right](\sin 21.340^\circ)\ \hat{k}$$

$$= -\ 200\ \frac{kg\ m^2}{s}\ \hat{k} . \tag{7}$$

10.) **Solution:**



In this "collision," the angular momentum is conserved as there is no net external torque. We can write

$$L_{JBC} = I_o\omega_o = L_{JAC} = I_f\omega_f \ . \qquad (1)$$

So,

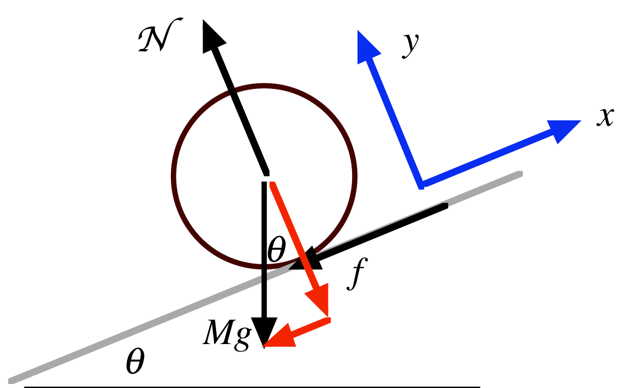
$$\omega_f = \frac{I_o}{I_f}\omega_o = \left[\frac{(1/2)MR^2}{(1/2)MR^2 + mr^2}\right]\omega_o$$

$$= \left[\frac{(1/2)(148\ kg)(10\ m)^2}{(1/2)(148\ kg)(10\ m)^2 + (25\ kg)((2/3)(10m))^2}\right](12\ rad/s)$$

$$= 10.4\ rad/s . \qquad (2)$$

11.) **Solution:**

a)



b) Summing forces parallel to the x-axis we have

$$Ma_x = -Mg\,sin\theta - f \; . \tag{1}$$

Summing the torques about the center of the hollow sphere we find

$$Rf = I\alpha = \left[(2/3)MR^2\right]\left[a_x / R\right] , \tag{2}$$

and

$$f = (2/3)Ma_x \; . \tag{3}$$

Substitution of equation (3) into equation (1) yields

$$Ma_x = -Mg\,sin\theta - (2/3)Ma_x \; , \tag{4}$$

and

$$\left[M + (2/3)M\right]a_x = -Mg\,sin\theta = (5/3)Ma_x \; . \tag{5}$$

So, the acceleration of the center of the hollow sphere relative to the incline is

$$a_x = -(3/5)g\,sin\theta \; . \tag{6}$$

The magnitude is

$$\left|a_x\right| = (3/5)g\,sin\theta = (3/5)(9.805\;m/s^2)(sin 22°) = 2.204\;m/s^2 . \tag{7}$$

c) The magnitude of the angular acceleration is

$$\left|\alpha\right| = \frac{\left|a_x\right|}{R} = \frac{(2.204\;m/s^2)}{(0.120\;m)} = 18.4\;\frac{rad}{s^2} \; . \tag{8}$$

d) The distance the hollow sphere moves before coming to rest is found using

$$0 = v_o^2 - 2\left|a_x\right|\ell \; . \tag{9}$$

So,

$$\ell = \frac{v_o^2}{2\left|a_x\right|} = \frac{(13.68\;m/s)^2}{2(2.204\;m/s^2)} = 42.5\;m . \tag{10}$$

e) We can write

$$f = \mu\mathcal{N} = \mu Mg\,cos\theta = \frac{2}{3}M\left|a_x\right| = \frac{2}{3}M\left[\frac{3}{5}g\,sin\theta\right] . \tag{11}$$

So,

$$\mu = \frac{(2/3)M\left[(3/5)g\,sin\theta\right]}{Mg\,cos\theta} = (2/5)tan\theta = 0.162 \; . \tag{12}$$

12.) Solution:

b) Summing the forces parallel to the x-axis we find

$$Mg\,sin\theta - f = Ma_x . \tag{1}$$

Summing the torques about the center of the thin hoop gives us

$$Rf = I\left|\alpha\right| = \left[MR^2\right]\left[a_x / R\right] , \tag{2}$$

and, therefore,

$$f = Ma_x \; . \tag{3}$$

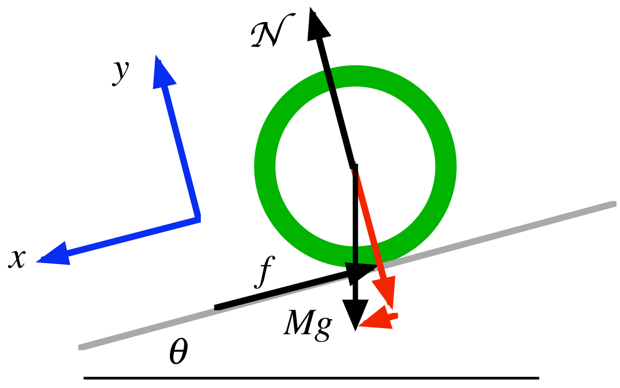
Substitution of equation (3) into a rearranged equation (1) yields

$$Mg\,sin\theta = Ma_x + Ma_x = 2Ma_x , \tag{4}$$

and the x acceleration is

$$a_x = (1/2)\, g \sin\theta = (1/2)\left(9.805\ m/s^2\right)\left(\sin 15°\right) = 1.269\ m/s^2\ . \qquad (5)$$

a)



c) The magnitude of the angular acceleration is

$$|\alpha| = \frac{a_x}{R} = \frac{\left(1.269\ m/s^2\right)}{\left(0.160\ m\right)} = 7.930\ \frac{rad}{s^2}\ . \qquad (6)$$

d) As the acceleration is constant and the initial speed is zero, we have

$$v = \sqrt{2a_x \ell} = \sqrt{2\left(1.269\ m/s^2\right)\left(1.500\ m\right)} = 1.951\ m/s\ . \qquad (7)$$

e) For the coefficient of rolling friction--a static friction--we write

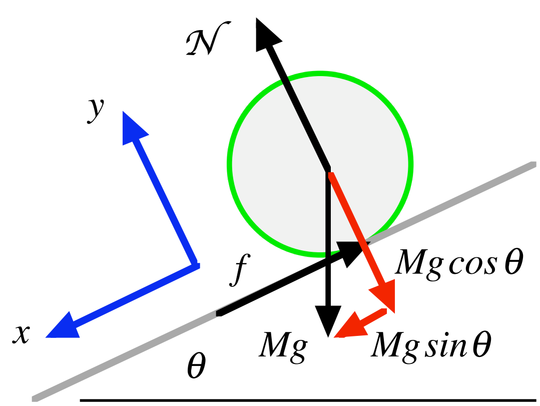
$$f = \mu \mathcal{N} = \mu\left(Mg\cos\theta\right) = Ma_x = M\left(\frac{1}{2} g \sin\theta\right), \qquad (8)$$

and, therefore,

$$\mu = \frac{M\left[(1/2)\, g \sin\theta\right]}{\left(Mg\cos\theta\right)} = \frac{1}{2}\tan\theta = \frac{1}{2}\left(\tan 15°\right) = 0.1340\ . \qquad (8)$$

13.) Solution:

a)



b) Summing the forces parallel to the x-axis we find

$$Mg\sin\theta - f = Ma_x \,. \tag{1}$$

Summing the torques about the center of the cylinder gives us

$$Rf = I|\alpha| = \left[(1/2)MR^2\right]\left[a_x / R\right] \,, \tag{2}$$

and, therefore,

$$f = (1/2)Ma_x \,. \tag{3}$$

Substitution of equation (3) into a rearranged equation (1) yields

$$Mg\sin\theta = Ma_x + (1/2)Ma_x = (3/2)Ma_x \,, \tag{4}$$

and the x acceleration is

$$a_x = (2/3)g\sin\theta = (2/3)(9.805\ m/s^2)(\sin 25°) = 2.763\ m/s^2 \,. \tag{5}$$

c) The magnitude of the angular acceleration is

$$|\alpha| = \frac{a_x}{R} = \frac{(2.763\ m/s^2)}{(0.558\ m)} = 4.952\ \frac{rad}{s^2} \,. \tag{6}$$

d) As the acceleration is constant and the initial speed is zero, we have

$$v = \sqrt{2a_x\ell} = \sqrt{2(2.763\ m/s^2)(1.500\ m)} = 2.879\ m/s \,. \tag{7}$$

e) For the coefficient of rolling friction--a static friction--we write

$$f = \mu\mathcal{N} = \mu(Mg\cos\theta) = \frac{1}{2}Ma_x = \frac{1}{2}M\left(\frac{2}{3}g\sin\theta\right) , \tag{8}$$

and, therefore,

$$\mu = \frac{M\left[(1/3)g\sin\theta\right]}{(Mg\cos\theta)} = \frac{1}{3}\tan\theta = \frac{1}{3}(\tan 25°) = 0.155 \,. \tag{8}$$

14.) Solution:

Maximum radius: r_{max}

The radius will be largest if the motion of the block on the turntable has motion impending in a sense away from the center. So the static friction would be a maximum and pointed toward the center of the circular path. Using the free-body diagram below, we write for the hanging block:

$$T - mg = ma_{vert} = 0 \,, \tag{1}$$

and

$$T = mg \,. \tag{2}$$

For the rotating block

$$\mathcal{N} - Mg - T\sin\varphi = Ma_{vert} = 0 \,, \tag{3}$$

and

$$\mathcal{N} = Mg + T\sin\varphi = Mg + mg\sin\varphi = \left[M + m\sin\varphi\right]g \,. \tag{4}$$

As motion is impending,

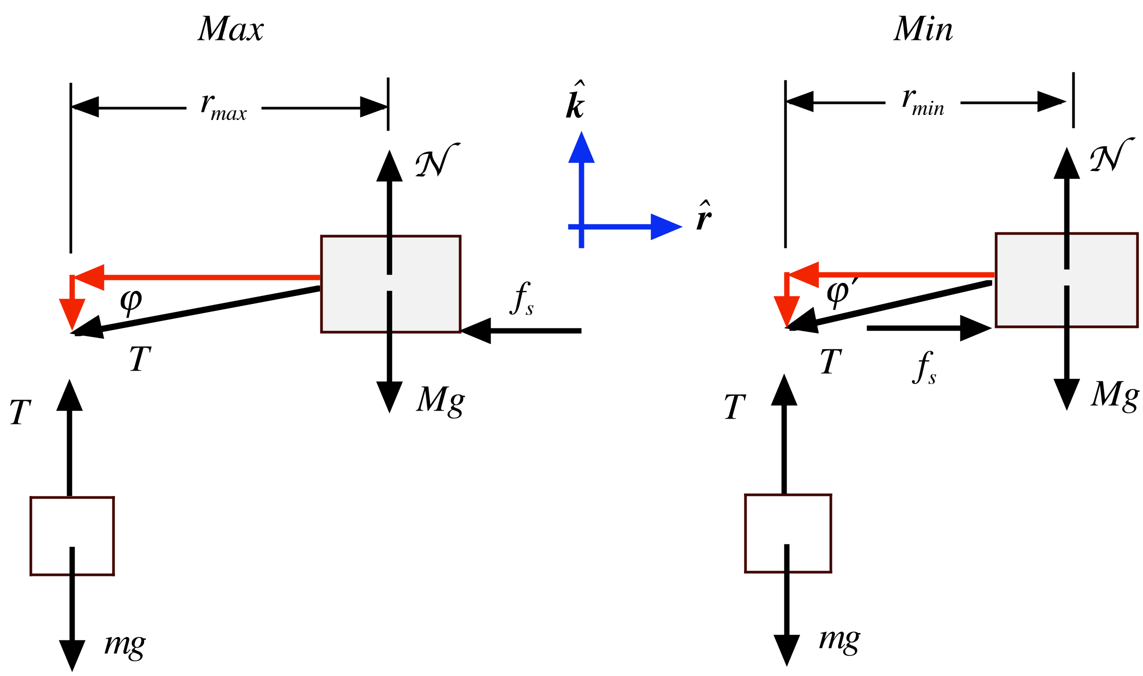
$$f_s = \mu\mathcal{N} = \mu\left[M + m\sin\varphi\right]g \,. \tag{5}$$

Summing the radially directed forces we have

$$-T\cos\varphi - \mu\left[M + m\sin\varphi\right]g = -Mr_{max}\omega^2 = -mg\cos\varphi - \mu\left[M + m\sin\varphi\right]g \,. \tag{6}$$

Solving for r_{max} we get

$$r_{max} = \left[\frac{m}{M} (\cos\varphi + \mu \sin\varphi) + \mu \right] \frac{g}{\omega^2} \ . \tag{7}$$



Minimum radius: r_{min}

In this case, the rotating block has impending motion toward the center and the static friction points away fro the center. Summing the radial forces on the rotating block, we have

$$-T\cos\varphi + \mu[M + m\sin\varphi']g = -Mr_{min}\omega^2 = -mg\cos\varphi' + \mu[M + m\sin\varphi']g \ . \tag{8}$$

Solving for r_{min} we get

$$r_{min} = \left[\frac{m}{M} (\cos\varphi' - \mu \sin\varphi') - \mu \right] \frac{g}{\omega^2} \ . \tag{9}$$

Note: If $\varphi = \varphi' = 0$, then

$$r_{max} = \left[\frac{m}{M} + \mu \right] \frac{g}{\omega^2} \ , \tag{10}$$

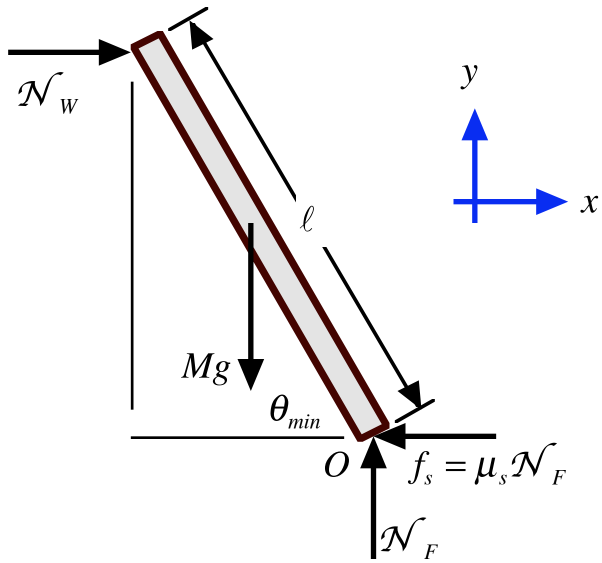
and

$$r_{min} = \left[\frac{m}{M} - \mu \right] \frac{g}{\omega^2} \ . \tag{11}$$

Solutions to Problems for Chapter 15

1.) Solution:

a)



b)

Equilibrium requires:

$$\sum \vec{F} = \sum \vec{\Gamma}_{any\ axis} = 0 \ . \tag{1}$$

So, the forces directed $\rightarrow$ must equal in magnitude the forces directed $\leftarrow$, and, therefore,

$$\mathcal{N}_W = \mu_s \mathcal{N}_F \ . \tag{2}$$

Also, the forces directed $\uparrow$ must equal in magnitude the forces directed $\downarrow$, and, therefore,

$$\mathcal{N}_F = Mg \ . \tag{3}$$

Using equations (2) and (3), we write

$$\mathcal{N}_W = \mu_s Mg \ . \tag{4}$$

Summing torques about point O, the magnitude of the clockwise torques must equal that of the counterclockwise torques. So, we can write

$$\left(\ell \sin\theta_{min}\right)\left(\mu_s Mg\right) = \left[\left(\ell/2\right)\cos\theta_{min}\right]\left(Mg\right) \ . \tag{5}$$

Using equation (5) we can now write

$$\frac{\ell \sin\theta_{min}}{\ell \cos\theta_{min}} = \frac{Mg}{2\mu_s Mg} = \tan\theta_{min} = \frac{1}{2\mu_s} \ , \tag{6}$$

and, therefore,

$$\theta_{min} = \tan^{-1}\left[\frac{1}{2\mu_s}\right] = \tan^{-1}\left[\frac{1}{2(0.426)}\right] = 49.6° \ . \tag{7}$$

c) The forces directed $\rightarrow$ must equal in magnitude the forces directed $\leftarrow$. So,

$$\mathcal{N}_W = \mu_s \mathcal{N}_F \ . \tag{8}$$

The forces directed $\uparrow$ must equal in magnitude the forces directed $\downarrow$. Therefore,

$$\mathcal{N}_F = Mg + C_1Mg = (1 + C_1)Mg \ . \tag{9}$$

Using equations (8) and (9), we write

$$\mathcal{N}_W = \mu_s (1 + C_1) Mg \ . \tag{10}$$

Summing torques about point O, the magnitude of the clockwise torques must equal that of the counterclockwise torques. So, we can write

$$(\ell \sin\theta_{min})(\mu_s (1 + C_1)Mg) = ((\ell/2)\cos\theta_{min})(Mg) + (C_2\ell\cos\theta_{min})(C_1Mg) \ . \tag{11}$$

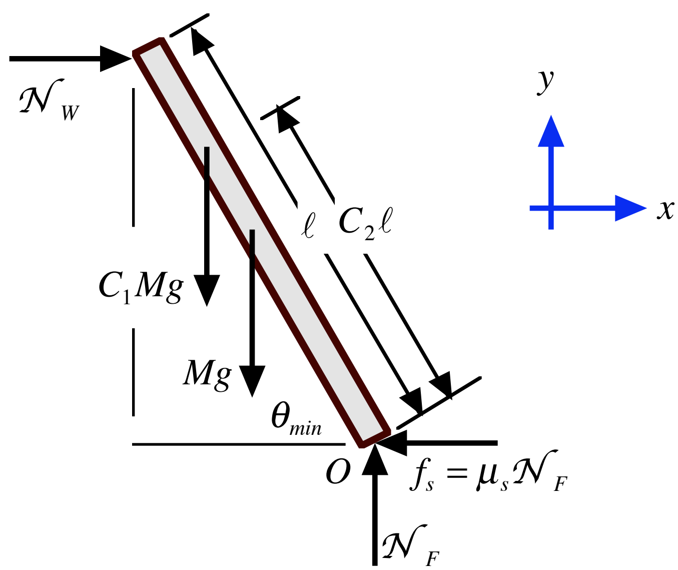
We can now write

$$\frac{\sin\theta_{min}}{\cos\theta_{min}} = \tan\theta_{min} = \frac{(1/2) + (C_1C_2)}{\mu_s (1 + C_1)} \ , \tag{12}$$

and, therefore,

$$\theta_{min} = \tan^{-1}\left[\frac{(1/2) + (C_1C_2)}{\mu_s (1 + C_1)}\right] . \tag{13}$$

d) $$\theta_{min} = \tan^{-1}\left[\frac{(1/2) + ((2)(1/3))}{(.426)(1 + (2))}\right] = 42.4° \ . \tag{14}$$



2.) Solution:

Equilibrium requires:

$$\sum \vec{F} = \sum \vec{\Gamma}_{any\ axis} = 0 \ . \tag{1}$$

b) For the tension in the cable supporting the load mass, we write

$$T_L - M_Lg = M_La_L = 0 \ . \tag{2}$$

and

$$T_L = M_Lg = (360\ kg)(9.805\ m/s^2) = 3{,}530\ N \ . \tag{3}$$

c) and d) For the beam, the forces directed $\rightarrow$ must equal in magnitude the forces directed $\leftarrow$,

and, therefore,

$$P_x = T\cos\varphi \ . \tag{4}$$

Also, the forces directed $\uparrow$ must equal in magnitude the forces directed $\downarrow$, and, therefore,

$$P_y = M_b g + T_L + T\sin\varphi = \left(M_b + M_L\right)g + T\sin\varphi \ . \tag{5}$$

Summing torques about point O, the magnitude of the clockwise torques must equal that of the counterclockwise torques. So, we can write

$$\left(\left(\ell/2\right)\cos\theta\right)\left(M_b g\right) + \left(\left(4\ell/5\right)\cos\theta\right)\left(M_L g\right) = \left(\ell\sin\left(\theta-\varphi\right)\right)\left(T\right) \ . \tag{6}$$

$$T = \frac{\left[\left(1/2\right)M_b + \left(4/5\right)M_L\right]g\cos\theta}{\sin\left(\theta-\varphi\right)}$$

$$= \frac{\left[\left(1/2\right)\left(80\ kg\right) + \left(4/5\right)\left(360\ kg\right)\right]\left(9.805\ m/s^2\right)\left(\cos 65°\right)}{\sin\left(\left(65°\right)-\left(35°\right)\right)} = 2{,}718\ N \ . \tag{7}$$

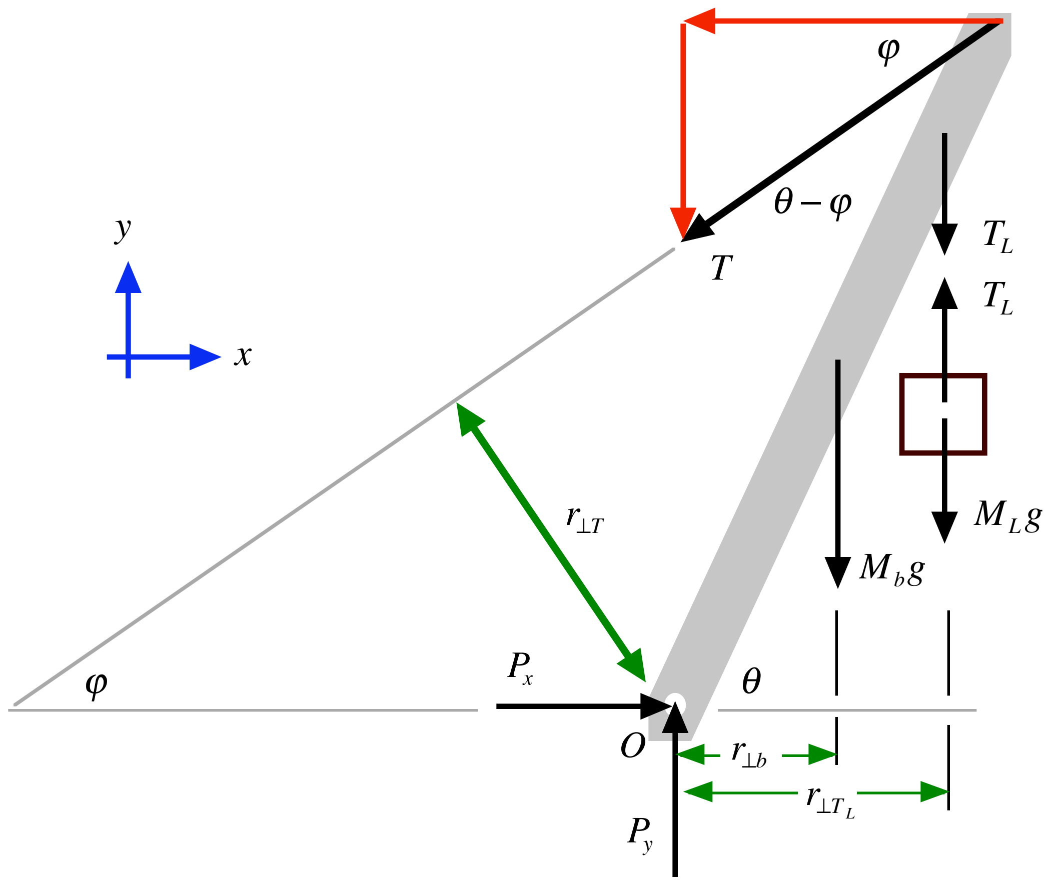
Using equation (4), we find

$$P_x = \left(2{,}718\ N\right)\left(\cos 35°\right) = 2{,}227\ N \ . \tag{8}$$

Equation (5) yields

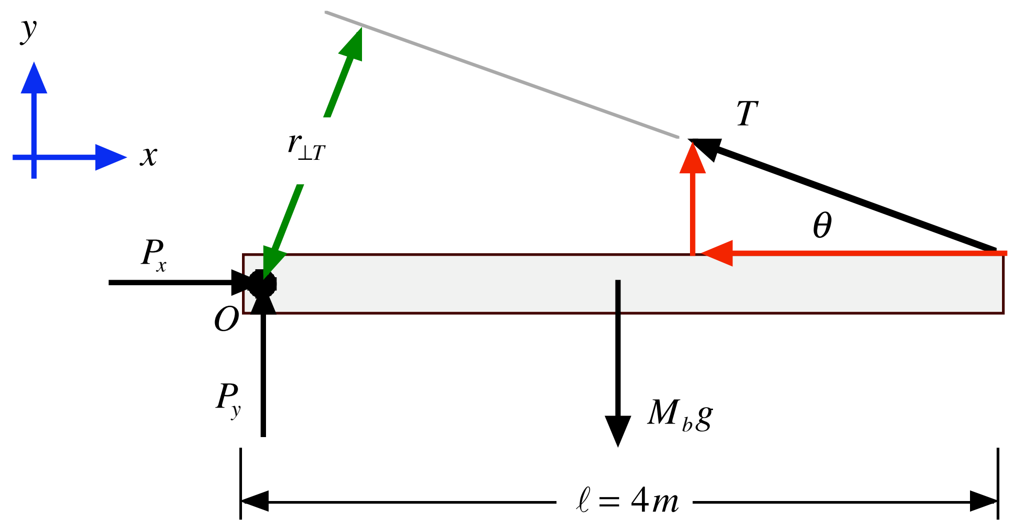
$$P_y = \left(\left(80\ kg\right) + \left(360\ kg\right)\right)\left(9.805\ m/s^2\right) + \left(2{,}718\ N\right)\left(\sin 35°\right) = 5{,}873\ N \ . \tag{9}$$

a)



3.) Solution:

a)



b) and c)

The forces directed $\rightarrow$ must equal in magnitude the forces directed $\leftarrow$. So,

$$P_x = T\cos\theta \ . \tag{1}$$

Also, the forces directed $\uparrow$ must equal in magnitude the forces directed $\downarrow$. So,

$$P_y + T\sin\theta = M_b g \ , \tag{2}$$

and

$$P_y = M_b g - T\sin\theta \ . \tag{3}$$

Summing torques about point O, the magnitude of the clockwise torques must equal that of the counterclockwise torques. So, we can write

$$(\ell/2)(M_b g) = (\ell\sin\theta)(T) \ , \tag{4}$$

and

$$T = \frac{(1/2)M_b g}{\sin\theta} = \frac{(1/2)(80 \ kg)(9.805 \ m/s^2)}{(\sin 20°)} = 1{,}147 \ N \ . \tag{5}$$

So, the horizontal force exerted on the beam by the pin is given by

$$P_x = (1{,}147 \ N)(\cos 20°) = 1{,}078 \ N \ , \tag{6}$$

while the vertical force exerted on the beam by the pin is

$$P_y = M_b g - T\sin\theta = (80 \ kg)(9.805 \ m/s^2) - (1{,}147 \ N)(\sin 20°) = 392 \ N \ . \tag{7}$$

4.) Solution:

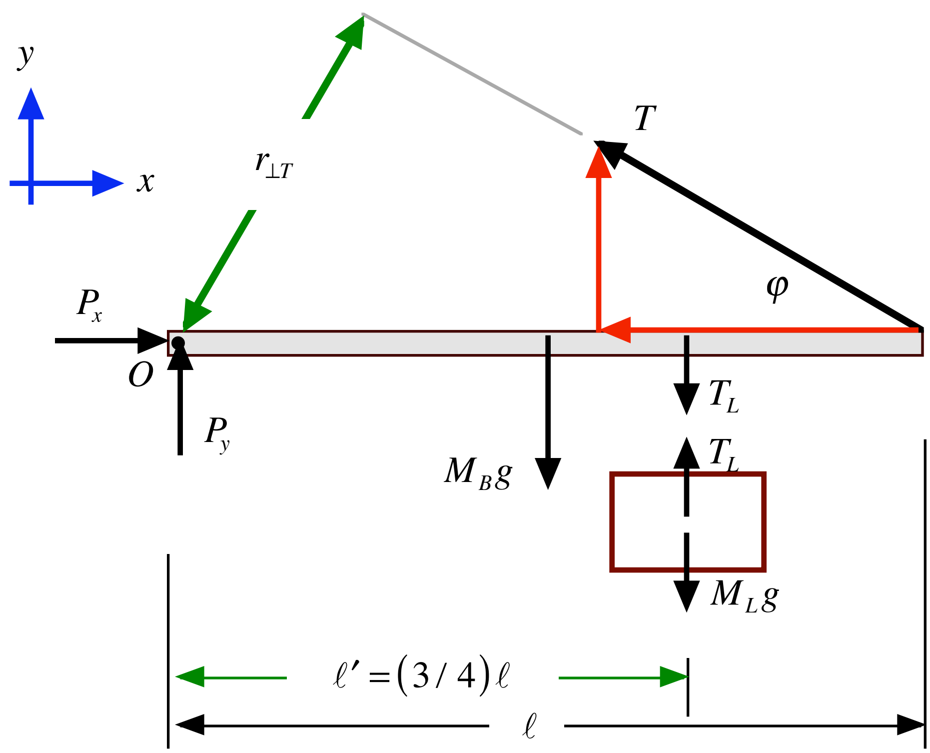
b) We can write for the tension in the cable attached to the load

$$T_L - M_L g = M_L a_L = 0 \ , \tag{1}$$

and

$$T_L = M_L g = (500 \ kg)(9.805 \ m/s^2) = 4{,}903 \ N \ . \tag{2}$$

a)



c) and d) The forces directed $\rightarrow$ must equal in magnitude the forces directed $\leftarrow$. So,

$$P_x = T\cos\varphi \ . \tag{3}$$

Also, the forces directed $\uparrow$ must equal in magnitude the forces directed $\downarrow$. So,

$$P_y + T\sin\varphi = M_B g + T_L \ , \tag{4}$$

and

$$P_y = (M_B + M_L)g - T\sin\varphi \ . \tag{5}$$

Summing torques about point O, the magnitude of the clockwise torques must equal that of the counterclockwise torques. So, we can write

$$(\ell/2)(M_B g) + (3\ell/4)(M_L g) = (\ell\sin\varphi)(T) \ , \tag{6}$$

and

$$T = \frac{[(1/2)M_B + (3/4)M_L]g}{\sin\varphi}$$

$$= \frac{[(1/2)(55\ kg) + (3/4)(500\ kg)](9.805\ m/s^2)}{(\sin 30°)} = 7{,}893\ N \ . \tag{7}$$

So, the horizontal force exerted on the beam by the pin is given by

$$P_x = (7{,}893\ N)(\cos 30°) = 6{,}836\ N \ , \tag{8}$$

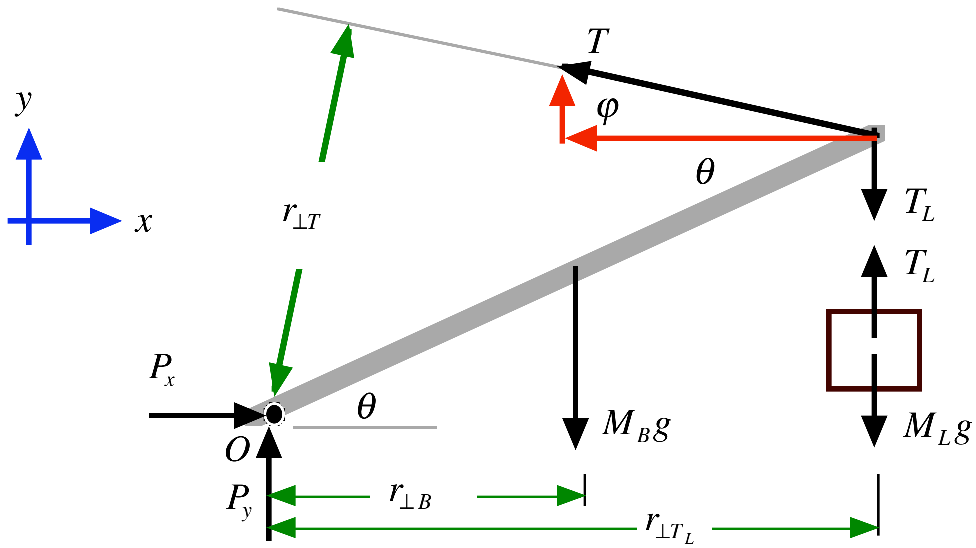
while the vertical force exerted on the beam by the pin is

$$P_y = (M_B + M_L)g - T\sin\varphi$$

$$= ((55\ kg) + (500\ kg))(9.805\ m/s^2) - (7{,}893\ N)(\sin 30°) = 1{,}495\ N \ . \tag{9}$$

5.) Solution:

a)



b) We can write for the tension in the cable attached to the load

$$T_L - M_L g = M_L a_L = 0 \ , \tag{1}$$

and

$$T_L = M_L g = \left(6\left(25 \ kg\right)\right)\left(9.805 \ m/s^2\right) = 1{,}471 \ N \ . \tag{2}$$

c) and d) The forces directed $\longrightarrow$ must equal in magnitude the forces directed $\longleftarrow$. So,

$$P_x = T \cos\varphi \ . \tag{3}$$

Also, the forces directed $\uparrow$ must equal in magnitude the forces directed $\downarrow$. So,

$$P_y + T \sin\varphi = M_B g + T_L \ , \tag{4}$$

and

$$P_y = \left(M_B + M_L\right) g - T \sin\varphi = 7 M_B g - T \sin\varphi \ . \tag{5}$$

Summing torques about point O, the magnitude of the clockwise torques must equal that of the counterclockwise torques. So, we can write

$$\left(\left(\ell/2\right)\cos\theta\right)\left(M_B g\right) + \left(\ell\cos\theta\right)\left(6 M_B g\right) = \left(\ell \sin\left(\varphi+\theta\right)\right)\left(T\right) \ , \tag{6}$$

and

$$T = \frac{(6.5) M_B g \cos\theta}{\sin\left(\varphi+\theta\right)}$$

$$= \frac{(6.5)\left(25 \ kg\right)\left(9.805 \ m/s^2\right)\left(\cos 25°\right)}{\left(\sin 37.5°\right)} = 2{,}372 \ N \ . \tag{7}$$

So, the horizontal force exerted on the beam by the pin is given by

$$P_x = \left(2{,}372 \ N\right)\left(\cos 12.5°\right) = 2{,}316 \ N \ , \tag{8}$$

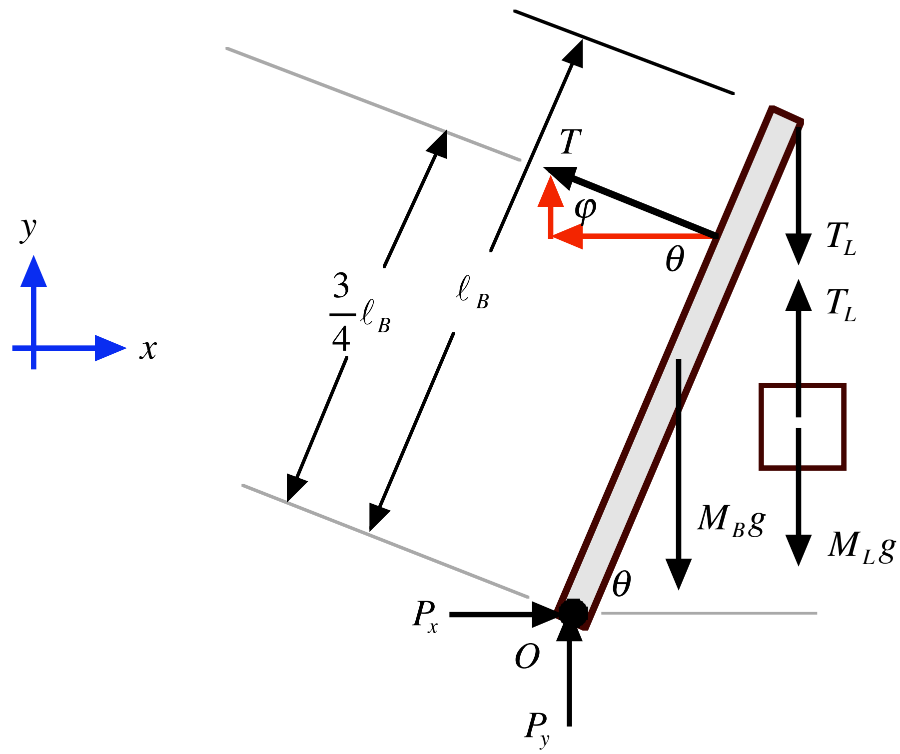
while the vertical force exerted on the beam by the pin is

$$P_y = 7 M_B g - T \sin\varphi$$

$$= 7\left(25 \ kg\right)\left(9.805 \ m/s^2\right) - \left(2{,}372 \ N\right)\left(\sin 12.5°\right) = 1{,}203 \ N \ . \tag{9}$$

6.) Solution:

a)



b) We can write for the tension in the cable attached to the load

$$T_L - M_L g = M_L a_L = 0 \ , \tag{1}$$

and

$$T_L = M_L g = (40 \ kg)(9.805 \ m/s^2) = 392 \ N \ . \tag{2}$$

c) and d) The forces directed $\rightarrow$ must equal in magnitude the forces directed $\leftarrow$. So,

$$P_x = T \cos\varphi \ . \tag{3}$$

Also, the forces directed $\uparrow$ must equal in magnitude the forces directed $\downarrow$. So,

$$P_y + T \sin\varphi = M_B g + T_L \ , \tag{4}$$

and

$$P_y = (M_B + M_L) g - T \sin\varphi \ . \tag{5}$$

Summing torques about point O, the magnitude of the clockwise torques must equal that of the counterclockwise torques. So, we can write

$$\big((\ell_B/2)\cos\theta\big)(M_B g) + (\ell_B \cos\theta)(M_L g) = \big((3/4)\ell_B\big)(T) \ , \tag{6}$$

and

$$T = \frac{\left[(1/2)M_B + M_L\right] g \cos\theta}{(3/4)}$$

$$= \frac{\left[(1/2)(20 \ kg) + (40 \ kg)\right](9.805 \ m/s^2)(\cos 60°)}{(3/4)} = 327 \ N \ . \tag{7}$$

So, the horizontal force exerted on the beam by the pin is given by

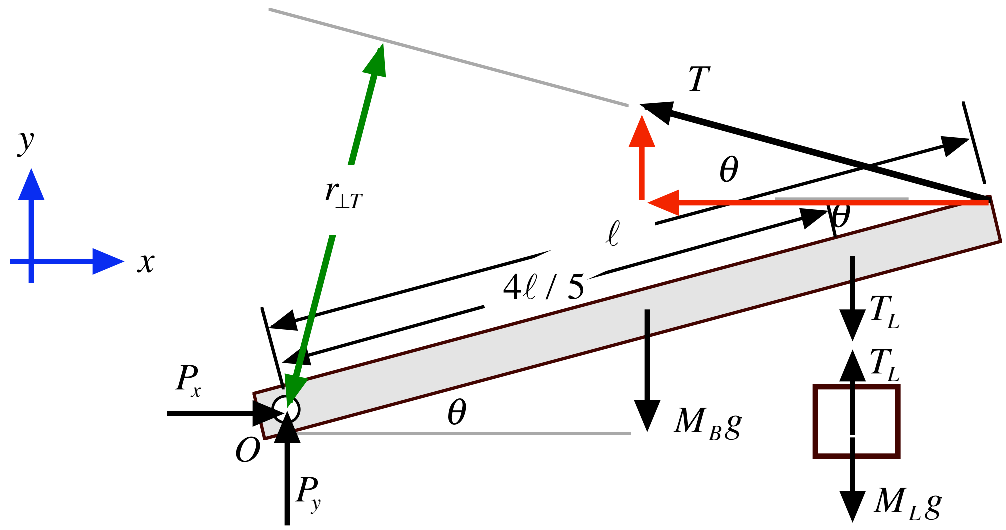
$$P_x = (327\ N)(cos\,30°) = 283\ N\ , \qquad (8)$$

while the vertical force exerted on the beam by the pin is

$$P_y = (M_B + M_L)\,g - T\,sin\varphi$$
$$= ((20\ kg)+(40\ kg))(9.805\ m/s^2) - (327\ N)(sin\,30°) = 425\ N\ . \qquad (9)$$

7.) Solution:

a)



b) We can write for the tension in the cable attached to the load

$$T_L - M_L g = M_L a_L = 0\ , \qquad (1)$$

and

$$T_L = M_L g = (544\ kg)(9.805\ m/s^2) = 5{,}334\ N\ . \qquad (2)$$

c) and d) The forces directed $\longrightarrow$ must equal in magnitude the forces directed $\longleftarrow$. So,

$$P_x = T\,cos\,\theta\ . \qquad (3)$$

Also, the forces directed $\uparrow$ must equal in magnitude the forces directed $\downarrow$. So,

$$P_y + T\,sin\theta = M_B g + T_L\ , \qquad (4)$$

and

$$P_y = (M_B + M_L)\,g - T\,sin\theta\ . \qquad (5)$$

Summing torques about point O, the magnitude of the clockwise torques must equal that of the counterclockwise torques. So, we can write

$$((\ell/2)cos\,\theta)(M_B g) + ((4\ell/5)cos\theta)(M_L g) = (\ell\,sin\,2\theta)(T)\ , \qquad (6)$$

and

$$T = \frac{[(1/2)M_B + (4/5)M_L]\,g\,cos\theta}{(sin\,2\theta)}$$

$$= \frac{[(1/2)(255\ kg) + (4/5)(544\ kg)](9.805\ m/s^2)(cos\,15°)}{(sin(2(15°)))}$$

$$T = 10{,}660\ N\ . \tag{7}$$

So, the horizontal force exerted on the beam by the pin is given by

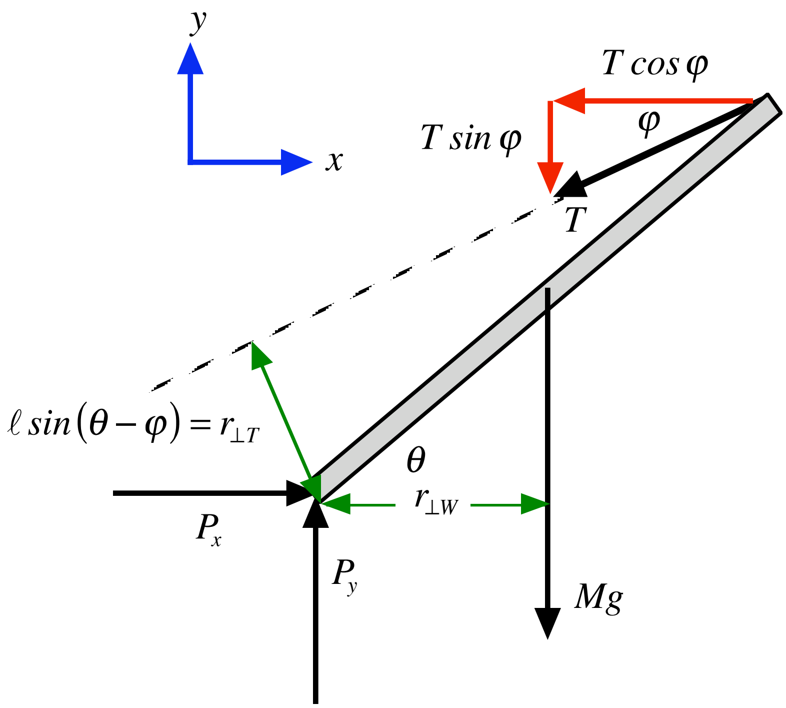
$$P_x = (10{,}660\ N)(\cos 15°) = 10{,}300\ N\ , \tag{8}$$

while the vertical force exerted on the beam by the pin is

$$P_y = (M_B + M_L)\,g - T\sin\theta$$

$$= (255\ kg + 544\ kg)(9.805\ m/s^2) - (10{,}660\ N)(\sin 15°) = 5{,}075\ N\ . \tag{9}$$

8.)



Static equilibrium requires

$$\rightarrow\ =\ \leftarrow$$

$$P_x = T\cos\varphi\ , \tag{1}$$

and

$$\uparrow\ =\ \downarrow$$

$$P_y = Mg + T\sin\varphi\ . \tag{2}$$

The torques with respect to the pin must equal zero, and , therefore,

$$\curvearrowright\ =\ \curvearrowleft$$

$$\left(\frac{\ell}{2}\cos\theta\right)(Mg) = (\ell\sin(\theta-\varphi))(T)\ . \tag{3}$$

a) and d): Using equation (3), we write

$$T = \frac{Mg\cos\theta}{2\sin(\theta-\varphi)} = \frac{(125\ kg)(9.8\ m/s^2)(\cos 40°)}{2\sin 10°} = 2{,}702\ N\ . \tag{4}$$

b) and d): Using equations (1) and (4), we can write

$$P_x = T\cos\varphi = \left[\frac{Mg\cos\theta}{2\sin(\theta-\varphi)}\right]\cos\varphi = \frac{(125\ kg)(9.8\ m/s^2)(\cos 40°)(\cos 30°)}{2\sin 10°}$$

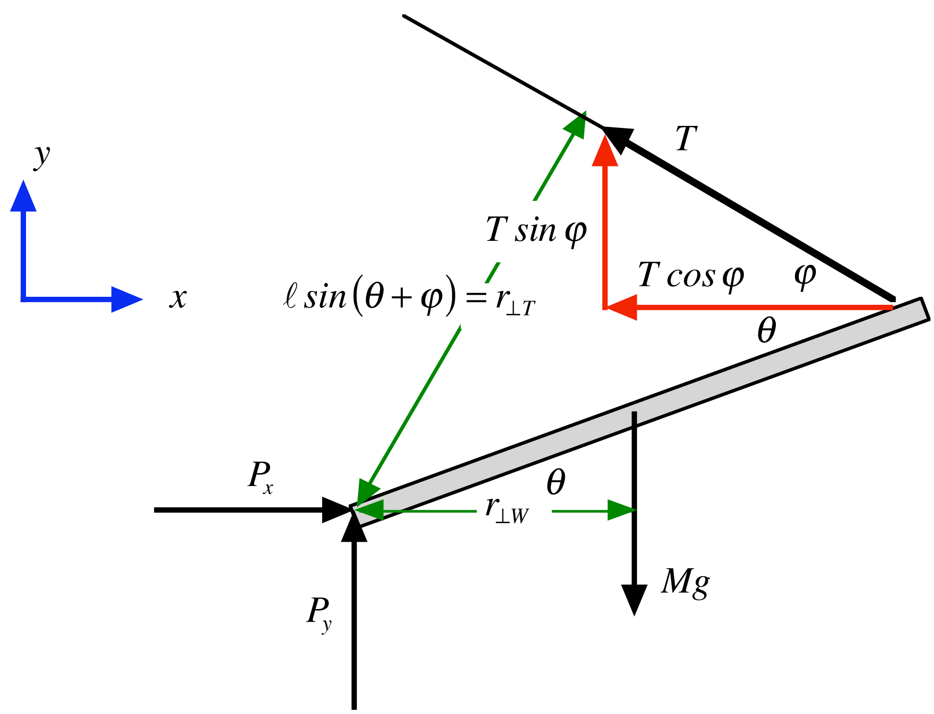
$$= 2,340\ N\ . \qquad (5)$$

c) and d): Using equations (2) and (4) yields

$$P_y = Mg + T\sin\varphi = Mg + \left[\frac{Mg\cos\theta}{2\sin(\theta-\varphi)}\right]\sin\varphi = Mg\left[1+\frac{\cos\theta\sin\varphi}{2\sin(\theta-\varphi)}\right]$$

$$= (125\ kg)(9.8\ m/s^2)\left[1+\frac{(\cos 40°)(\sin 30°)}{2\sin 10°}\right] = 2,576\ N\ . \qquad (6)$$

9.)



Static equilibrium requires

$$\rightarrow\ =\ \leftarrow$$

$$P_x = T\cos\varphi\ , \qquad (1)$$

and

$$\uparrow\ =\ \downarrow$$

$$P_y = Mg - T\sin\varphi\ . \qquad (2)$$

The torques with respect to the pin must equal zero, and , therefore,

$$\curvearrowright\ =\ \curvearrowleft$$

$$\left(\frac{\ell}{2}\cos\theta\right)(Mg) = (\ell\sin(\theta+\varphi))(T)\ . \qquad (3)$$

a) and d): Using equation (3), we write

$$T = \frac{Mg\cos\theta}{2\sin(\theta+\varphi)} = \frac{(125\ kg)(9.8\ m/s^2)(\cos 20°)}{2\sin 50°} = 751\ N\ . \qquad (4)$$

b) and d): Using equations (1) and (4), we can write

$$P_x = T\cos\varphi = \left[\frac{Mg\cos\theta}{2\sin(\theta+\varphi)}\right]\cos\varphi = \frac{(125\ kg)(9.8\ m/s^2)(\cos 20°)(\cos 30°)}{2\sin 50°}$$

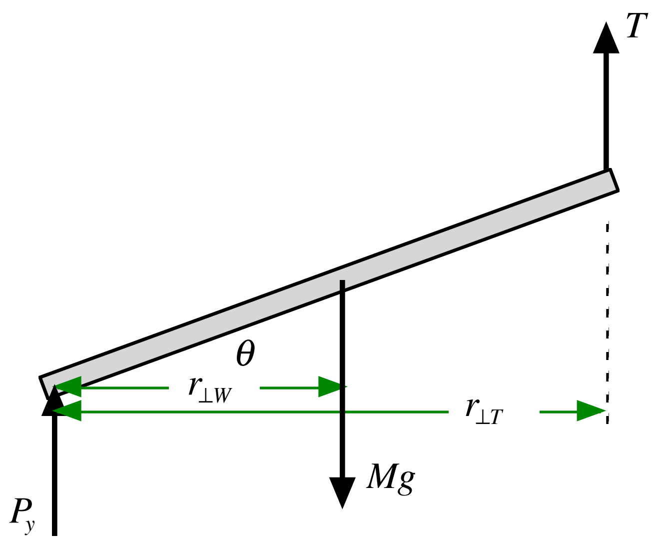
$$= 651\ N\ . \qquad (5)$$

c) and d): Using equations (2) and (4) yields

$$P_y = Mg - T\sin\varphi = Mg - \left[\frac{Mg\cos\theta}{2\sin(\theta+\varphi)}\right]\sin\varphi = Mg\left[1 - \frac{\cos\theta\sin\varphi}{2\sin(\theta+\varphi)}\right]$$

$$= (125\ kg)(9.8\ m/s^2)\left[1 - \frac{(\cos 20°)(\sin 30°)}{2\sin 50°}\right] = 849\ N\ . \qquad (6)$$

10.)



b) and d): Static equilibrium requires

$$\rightarrow\ =\ \leftarrow$$
$$P_x = 0\ , \qquad (1)$$

and

$$\uparrow\ =\ \downarrow$$
$$P_y = Mg - T\ . \qquad (2)$$

The torques with respect to the pin must equal zero, and , therefore,

$$\curvearrowright\ =\ \curvearrowleft$$
$$\left(\frac{\ell}{2}\cos\theta\right)(Mg) = (\ell\cos\theta)(T)\ . \qquad (3)$$

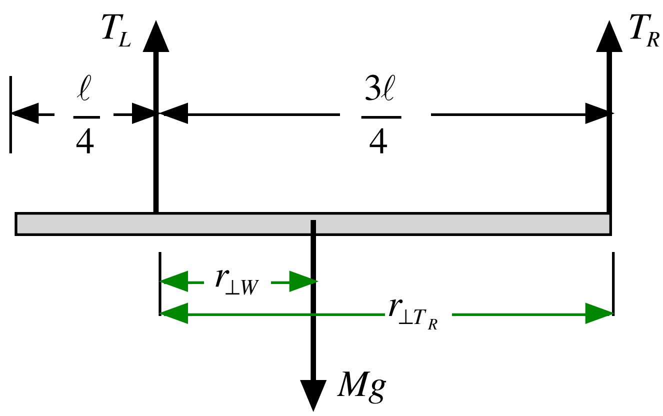
a) and d): Using equation (3), we write

$$T = \frac{Mg}{2} = \frac{(125\ kg)(9.8\ m/s^2)}{2} = 613\ N\ . \tag{4}$$

c) and d): Using equations (2) and (4) yields

$$P_y = Mg - T = Mg - \frac{Mg}{2} = \frac{Mg}{2} = \frac{(125\ kg)(9.8\ m/s^2)}{2} = 613\ N\ . \tag{5}$$

11.)



Static equilibrium requires

$$\uparrow\ =\ \downarrow$$
$$T_R = Mg - T_L\ , \tag{1}$$

while the torques about the point of attachment of T_L requires

$$r_{\perp W}\, Mg = r_{\perp T_R} T_R\ , \tag{2}$$

and,

$$\left(\frac{\ell}{2} - \frac{\ell}{4}\right) Mg = \left(\ell - \frac{\ell}{4}\right)(Mg - T_L)\ , \tag{3}$$

and

$$\frac{1}{3} Mg = Mg - T_L\ . \tag{4}$$

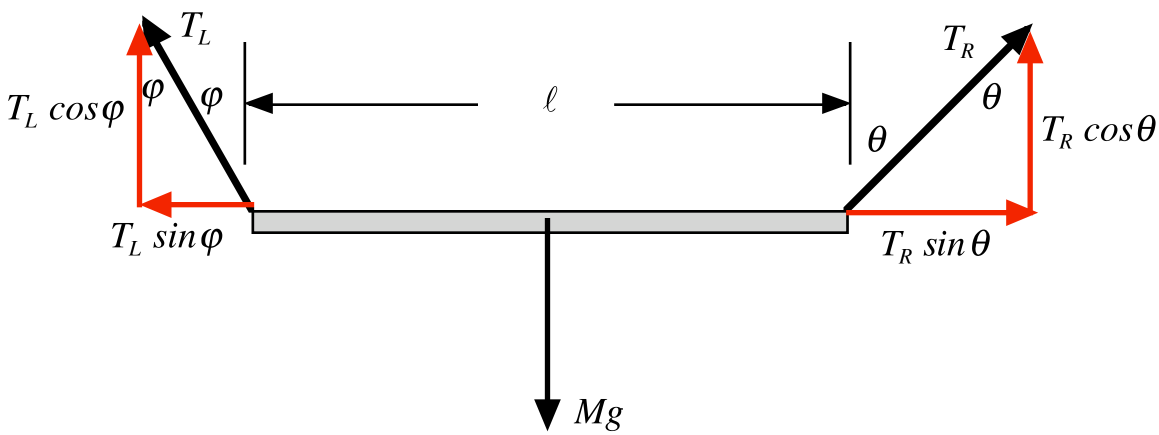
Therefore,

$$T_L = Mg - \frac{1}{3} Mg = \frac{2}{3} Mg = \frac{2}{3}(125\ kg)(9.8\ m/s^2) = 817\ N\ . \tag{5}$$

Equations (5) and (1) imply

$$T_R = Mg - T_L = Mg - \frac{2}{3} Mg = \frac{1}{3} Mg = \frac{1}{3}(125\ kg)(9.8\ m/s^2) = 408\ N\ . \tag{6}$$

12.)



a) Static equilibrium requires

$$\rightarrow\ =\ \leftarrow$$

$$T_R\ sin\,\theta = T_L\ sin\,\varphi\ , \tag{1}$$

and, therefore,

$$T_L = T_R\left[\frac{sin\theta}{sin\varphi}\right]\ . \tag{2}$$

Also, for static equilibrium, we have

$$\uparrow\ =\ \downarrow$$

$$T_L\ cos\varphi + T_R\,cos\,\theta = Mg\ . \tag{3}$$

Substitution of equation (2) into equation (3) yields

$$T_R\left[\frac{sin\,\theta}{sin\varphi}\right]cos\,\varphi + T_R\ cos\theta = Mg\ , \tag{4}$$

and

$$T_R\left[\frac{sin\theta}{tan\,\varphi} + cos\theta\right] = Mg\ , \tag{5}$$

and, therefore,

$$T_R = \frac{Mg}{\frac{sin\,\theta}{tan\varphi} + cos\,\theta} = \frac{Mg\,tan\,\varphi}{sin\,\theta + cos\theta\,tan\,\varphi}\ . \tag{6}$$

Substitution of equation (7) into equation (2) gives us

$$T_L = T_R\left[\frac{sin\theta}{sin\varphi}\right] = \left[\frac{Mg\ tan\,\varphi}{sin\,\theta + cos\,\theta\,tan\,\varphi}\right]\left[\frac{sin\,\theta}{sin\,\varphi}\right] = \frac{Mg}{cos\,\theta + \frac{tan\,\varphi}{tan\theta}}$$

$$= \frac{Mg\,tan\,\theta}{cos\,\varphi\,tan\,\theta + sin\,\varphi}\ . \tag{7}$$

b) Using equation (6), we have

$$T_R = \frac{(125\ kg)(9.8\ m/s^2)(tan\,30°)}{(sin\,45°)+(cos\,45°)(tan\,30°)} = 634.1\ N\ . \tag{8}$$

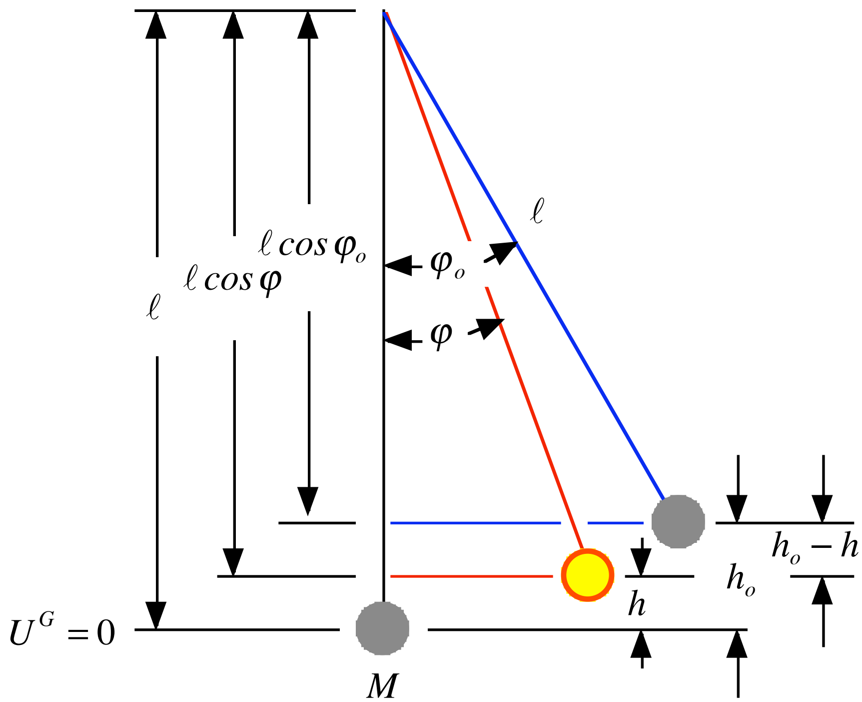
Using equation (7), we have

$$T_L = \frac{(125\ kg)(9.8\ m/s^2)(tan\,45°)}{(cos\,30°)(tan\,45°)+(sin\,30°)} = 896.8\ N\,. \tag{9}$$

Solutions to Problems for Chapter 16

1.) Solution:

a)



b) The tension is always directed perpendicular to the path of the bob so the tension does no work on the bob. The only force doing work on the bob is the conservative gravitational force. So we use the conservation of mechanical energy and can write

$$K_o + U_o^G = K_L + U_L^G \ , \tag{1}$$

and

$$0 + Mgh_o = \frac{1}{2}Mv^2 + Mgh \ , \tag{2}$$

where we assume the gravitational potential energy is zero when the bob is at its lowest point on the path. Solving for the speed, we have

$$v = \sqrt{2g\left(h_o - h\right)} \ . \tag{3}$$

Inspection of the diagram leads us to write

$$h_o - h = \ell\,cos\,\varphi - \ell\,cos\,\varphi_o = \ell\left[cos\,\varphi - cos\,\varphi_o\right] , \tag{4}$$

and

$$v = \sqrt{2g\ell\left(cos\,\varphi - cos\,\varphi_o\right)} \ . \tag{5}$$

At $\varphi = 15°$, we have

$$v\left(\varphi = 15°\right) = \sqrt{2\left(9.805 \ m/s^2\right)\left(1.25 \ m\right)\left(cos\,15° - cos\,30°\right)} = 1.56 \ m/s \ . \tag{6}$$

c) Maximum speed occurs when $\varphi = 0°$, when the bob is at the lowest point in its circular path. So,

$$v_{max} = \sqrt{2g\ell(1 - cos\varphi_o)}$$

$$= \sqrt{2(9.805\ m/s^2)(1.25\ m)(1 - cos\,30°)} = 1.812\ m/s\ . \qquad (7)$$

d) We can use equation (5) to determine how the angle is related to the speed. We write

$$v^2 = 2g\ell(cos\,\varphi - cos\varphi_o)\ , \qquad (8)$$

while

$$cos\,\varphi = \frac{v^2}{2g\ell} + cos\varphi_o\ , \qquad (9)$$

and

$$\varphi = cos^{-1}\left[\frac{v^2}{2g\ell} + cos\,\varphi_o\right]$$

$$= cos^{-1}\left\{\frac{[(1.812\ m/s)/3]^2}{2(9.805\ m/s^2)(1.25\ m)} + cos\,30°\right\} = 28.2°\ . \qquad (10)$$

Note: Assume at some angle φ the speed is given by

$$v = Cv_{max}\ . \qquad (11)$$

where C is a fraction for which $0 \le C \le 1$. Using equation (11) and the square of equations (5) and (7) gives us

$$v^2 = 2g\ell(cos\,\varphi - cos\varphi_o) = C^2 v_{max}^2 = C^2\left[2g\ell(1 - cos\,\varphi_o)\right], \qquad (12)$$

and

$$(cos\,\varphi - cos\,\varphi_o) = C^2(1 - cos\,\varphi_o)\ . \qquad (13)$$

Collecting terms, we find

$$cos\,\varphi = C^2 + cos\varphi_o - C^2\ cos\varphi_o = C^2 + (1 - C^2)cos\varphi_o\ . \qquad (14)$$

Finally, then, we have

$$\varphi = cos^{-1}\left[C^2 + (1 - C^2)cos\,\varphi_o\right]. \qquad (15)$$

Now for $C = 1/3$, we have

$$\varphi = cos^{-1}\left[(1/3)^2 + (1 - (1/3)^2)cos\,30°\right] = 28.2°\ . \qquad (16)$$

The bob has a third of its maximum speed after having moved only two *degrees*.

2.) Solution:

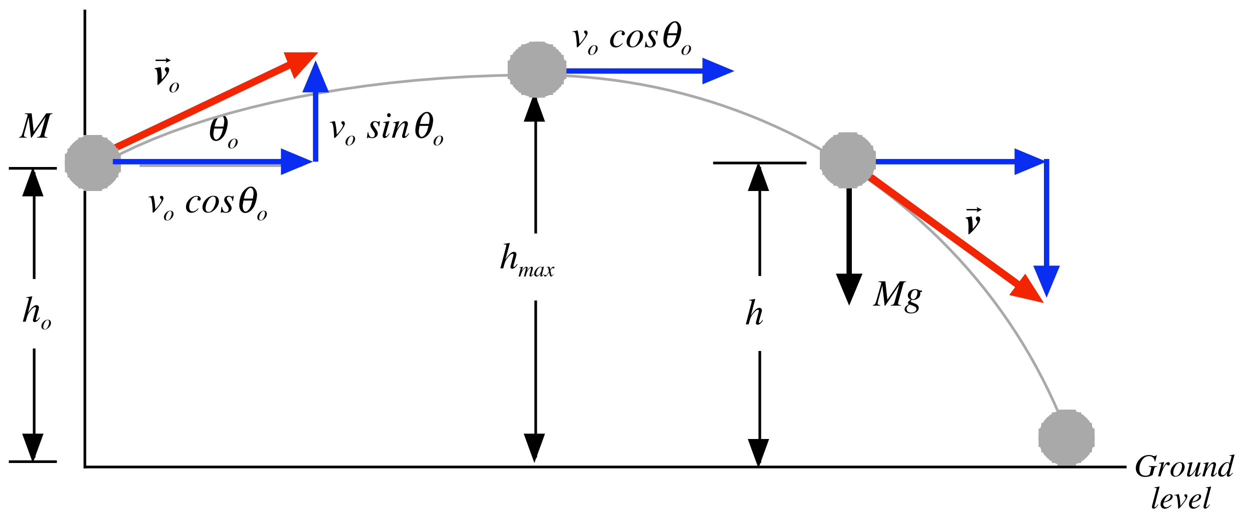
As we are ignoring air resistance, the only force acting on the sphere is the conservative gravitational force. So, we can use the conservation of mechanical energy. We let ground level represent the horizontal plane where the gravitational potential energy is zero. We have

$$K_o + U_o^G = K_L + U_L^G\ , \qquad (1)$$

$$\frac{1}{2}Mv_o^2 + Mgh_o = \frac{1}{2}Mv^2 + Mgh\ . \qquad (2)$$

Multiplying each term of equation (2) by 2 and dividing by M, we get

$$v_o^2 + 2gh_o = v^2 + 2gh \ . \tag{3}$$



c) At the "top" of its path, the vertical speed of the projectile is zero and the horizontal speed is

$$v_{top} = v_o \cos\theta_o \ . \tag{4}$$

This follows from the fact that there is no horizontal force to **change** the initial horizontal velocity.

a) To find the maximum height, we first solve equation (3) for h. We find

$$h = h_o + \frac{v_o^2 - v^2}{2g} \ . \tag{5}$$

So,

$$h_{max} = h_o + \frac{v_o^2 - (v_o \cos\theta_o)^2}{2g} = h_o + \frac{v_o^2 (1 - \cos^2\theta_o)}{2g}$$

$$= h_o + \frac{v_o^2 (\sin^2\theta_o)}{2g} = h_o + \frac{(v_o \sin\theta_o)^2}{2g}$$

$$= (30\ m) + \frac{[(15\ m/s)(\sin 25°)]^2}{2(9.805\ m/s^2)} = 32.05\ m \ . \tag{6}$$

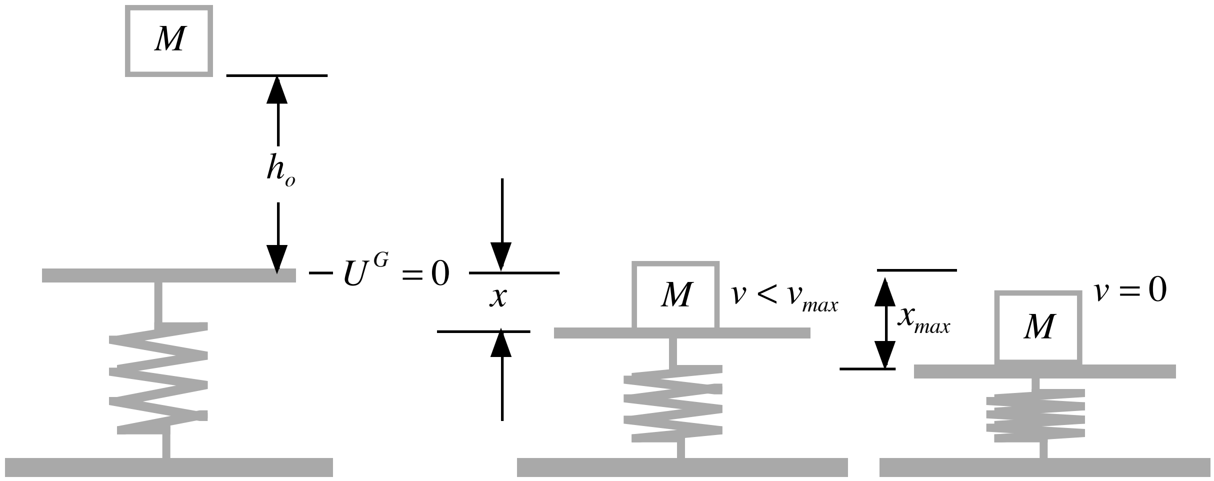
b) To find the speed of the sphere as it strikes the ground, we solve equation (3) for v. So

$$v = \sqrt{v_o^2 + 2g(h_o - h)} \ . \tag{7}$$

When the projectile strikes the ground, $h = 0$ and equation (7) becomes

$$v = \sqrt{v_o^2 + 2gh_o} = \sqrt{(15\ m/s)^2 + 2(9.805\ m/s^2)(30\ m)} = 28.5\ m/s \ . \tag{8}$$

3.) Solution:



The only forces that act on the block are the conservative gravitational force and the conservative elastic spring force. So, we can use the conservation of mechanical energy for this problem.

a) The block has its maximum speed at the instant it strikes the spring. So, during it fall, only the gravitational force is at work. We write

$$K_o + U_o^G = K_L + U_L^G \ , \tag{1}$$

$$0 + Mgh_o = \frac{1}{2}Mv_{max}^2 + 0 \ , \tag{2}$$

where I have chosen the gravitational potential energy to zero on the horizontal plane of the spring's equilibrium position. We have

$$v_{max} = \sqrt{2gh_o} = \sqrt{2\left(9.805\ m/s^2\right)\left(2.250\ m\right)} = 6.642\ m/s\ . \tag{3}$$

b) During the compression of the spring, the elastic spring force is also at work. So, we have, for some arbitrary compression distance x,

$$K_o + U_o^G + U_o^{sp} = K_L + U_L^G + U_L^{sp} \ , \tag{4}$$

$$0 + Mgh_o + 0 = \frac{1}{2}Mv^2 - Mgx + \frac{1}{2}k_{sp}x^2 \ . \tag{5}$$

To solve this equation for x, noting that it is quadratic in x, we collect all terms on one side of the equation and multiply each term by 2 and divide by k_{sp}. We get

$$x^2 - 2\left[\frac{Mg}{k_{sp}}\right]x + \left[\frac{M}{k_{sp}}\right]\left[v^2 - 2gh_o\right] = 0 \ . \tag{6}$$

During the compression of the spring, the speed of the block decreases. At any time in this process, the speed of the block is some fractional part of the maximum speed. So, we can write

$$v = fv_{max} = f\sqrt{2gh_o} \ , \tag{7}$$

where

$$0 \le f \le 1 \ . \tag{8}$$

Equation (7) implies

$$v^2 = f^2\left(2gh_o\right) . \tag{9}$$

Substitution of equation (9) into equation (6) gives us

$$x^2 - 2\left[\frac{Mg}{k_{sp}}\right]x + \frac{M}{k_{sp}}\left[f^2\left(2gh_o\right) - 2gh_o\right] = 0 \ , \tag{10}$$

and

$$x^2 - 2\left[\frac{Mg}{k_{sp}}\right]x - \frac{Mg}{k_{sp}}\left[2h_o\left(1 - f^2\right)\right] = 0 \ . \tag{11}$$

The roots of this equation are

$$x = \frac{Mg}{k_{sp}} \pm \sqrt{\left[\frac{Mg}{k_{sp}}\right]^2 + \frac{Mg}{k_{sp}}\left[2h_o\left(1 - f^2\right)\right]} \ . \tag{12}$$

As the magnitude of the square root term is greater than the first term, we can ignore the negative solution as it has no legitimate physical meaning. So, in general

$$x = \frac{Mg}{k_{sp}} + \sqrt{\left[\frac{Mg}{k_{sp}}\right]^2 + \frac{Mg}{k_{sp}}\left[2h_o\left(1 - f^2\right)\right]} \ . \tag{13}$$

At that point when the speed of the block is one half of its maximum value, $f = 1/2$, and we have

$$\begin{aligned} x &= \frac{Mg}{k_{sp}} + \sqrt{\left[\frac{Mg}{k_{sp}}\right]^2 + \frac{Mg}{k_{sp}}\left[2h_o\left(1 - (1/2)^2\right)\right]} \\ &= \frac{Mg}{k_{sp}} + \sqrt{\left[\frac{Mg}{k_{sp}}\right]^2 + \frac{Mg}{k_{sp}}\left[\frac{3}{2}h_o\right]} \\ &= C + \sqrt{C^2 + \frac{3}{2}h_o C} \ , \end{aligned} \tag{14}$$

where

$$C = \frac{Mg}{k_{sp}} = \frac{(2.500 \ kg)(9.805 \ m/s^2)}{(5000 \ N/m)} = 0.0049025 \ m \ , \tag{15}$$

and $h_o = 2.25$. So, this compression distance is

$$\begin{aligned} x &= (0.0049025 \ m) + \sqrt{(0.0049025 \ m)^2 + \frac{3}{2}(2.25 \ m)(0.0049025 \ m)} \\ &= 0.1336 \ m \equiv 13.36 \ cm \ . \end{aligned} \tag{16}$$

c) Of course, maximum compression occurs at that instant where the speed of the block is zero and, therefore, $f = 0$. This makes equation (14) become

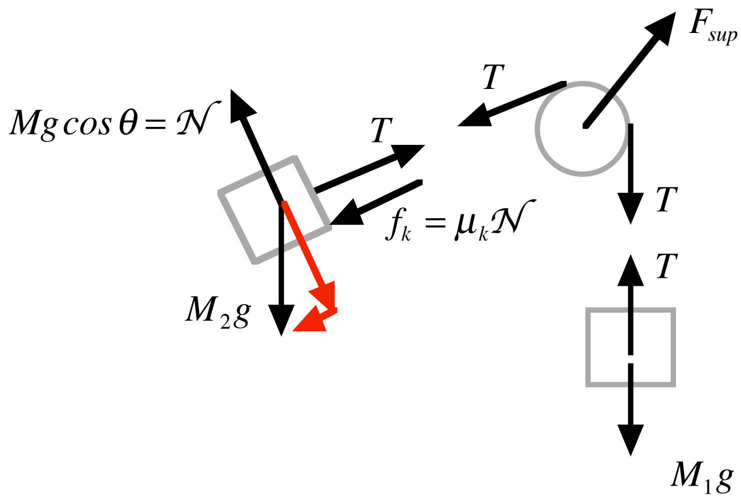
$$x_{max} = C + \sqrt{C^2 + 2h_o C}$$

$$= (0.0049025\ m) + \sqrt{(0.0049025\ m)^2 + 2(2.25\ m)(0.0049025\ m)}$$

$$= 0.1535\ m \equiv 15.35\ cm . \qquad (17)$$

The first half of the speed is lost in slightly more than 13 *cm* and the second half of the speed in only 2 *cm*.

4.) Solution:



The system we treat is made up of the two blocks, the pulley and the string. By including the string, we may ignore tension forces as they are "internal forces" and do no work. So, we can write

$$W_{\mathcal{N}} + W_G + W_{f_k} = K_L - K_o\ . \qquad (1)$$

a) As the normal force is constant, we can write

$$W_{\mathcal{N}} = (\mathcal{N})(\ell)(cos\,\angle_{bet}) = (M_2 g\,cos\,\theta)(\ell)(cos\,90°) = 0\ . \qquad (2)$$

b) The gravitational force is conservative so we can write

$$W_G = U_o^G - U_L^G\ . \qquad (3)$$

For simplicity, I set the later gravitational potential energy equal to zero. Initially, block one is above its zero point so it will have positive gravitational potential energy. Block two, however, is initially below its zero point so it will have negative potential energy. Therefore,

$$W_G = M_1 g\ell - M_2 g\ell\,sin\theta = (M_1 - M_2\,sin\,\theta)g\,\ell$$

$$= [(12\ kg) - (6\ kg)(sin\,25°)](9.805\ m/s^2)(1.25\ m) = 116\ Joules\ . \qquad (4)$$

c) The kinetic frictional force is also constant and we write

$$W_{f_k} = (f_k)(\ell)(cos\,\angle_{bet}) = (\mu_k M_2 g\,cos\,\theta)(\ell)(cos\,180°) = -\mu_k M_2 g\ell\,cos\,\theta$$

$$= -(0.2)(6\ kg)(9.805\ m/s^2)(1.25\ m)(cos\,25°) = -13.3\ Joules\,. \qquad (5)$$

d) As the system is released from rest, the initial kinetic energy is zero. We have for the later kinetic energy

$$K_L = \frac{1}{2}M_1 v^2 + \frac{1}{2}M_2 v^2 = \frac{1}{2}(M_1 + M_2)v^2 = \sum W\ . \qquad (6)$$

Therefore,

$$v = \sqrt{\frac{2\sum W}{(M_1 + M_2)}} = \sqrt{\frac{2(116\ J - 13.3\ J)}{(12\ kg + 6\ kg)}} = 3.38\ m/s\ . \tag{7}$$

5.) Solution:

a) The work done by the gravitational force is

$$W_G = U^G_{aphelion} - U^G_{perihelion}$$

$$= \left[-\frac{GM_\odot M_\oplus}{a_\oplus(1+\varepsilon_\oplus)}\right] - \left[-\frac{GM_\odot M_\oplus}{a_\oplus(1-\varepsilon_\oplus)}\right]$$

$$= \frac{GM_\odot M_\oplus}{a_\oplus}\left[\frac{1}{(1-\varepsilon_\oplus)} - \frac{1}{(1+\varepsilon_\oplus)}\right] = \left[\frac{2\varepsilon_\oplus}{\left(1-(\varepsilon_\oplus)^2\right)}\right]\frac{GM_\odot M_\oplus}{a_\oplus}$$

$$= \left[\frac{2(0.017)}{\left(1-(0.017)^2\right)}\right]\frac{(6.67\times10^{-11} Nm^2/kg^2)(1.99\times10^{30} kg)(5.98\times10^{24} kg)}{(1.49\times10^{11} m)}$$

$$= 1.80\times10^{32}\ Joules\ . \tag{1}$$

b) For circular motion,

$$\frac{GM_\odot M_\oplus}{d^2_{\odot\oplus}} = \frac{M_\oplus v^2}{d_{\odot\oplus}}\ , \tag{2}$$

and

$$v = \sqrt{\frac{GM_\odot}{d_{\odot\oplus}}}\ . \tag{3}$$

At perihelion, we have

$$v_{perihelion} = \sqrt{\frac{(6.67\times10^{-11} Nm^2/kg^2)(1.99\times10^{30} kg)}{(1-0.017)(1.49\times10^{11} m)}} = 30{,}040\ \frac{m}{s}\ . \tag{4}$$

At aphelion, we have

$$v_{aphelion} = \sqrt{\frac{(6.67\times10^{-11} Nm^2/kg^2)(1.99\times10^{30} kg)}{(1+0.017)(1.49\times10^{11} m)}} = 29{,}540\ \frac{m}{s}\ . \tag{5}$$

Note:

Comparison of equation (3) to that of the gravitational potential energy should convince you that

$$v = \sqrt{-\frac{U^G}{M_\oplus}}\ . \tag{6}$$

Squaring both sides of this equation we find

$$v^2 = -\frac{U^G}{M_\oplus}\ , \tag{7}$$

and, therefore,

$$\frac{1}{2}M_\oplus v^2 = K = -\frac{1}{2}U^G\ . \tag{8}$$

This implies that

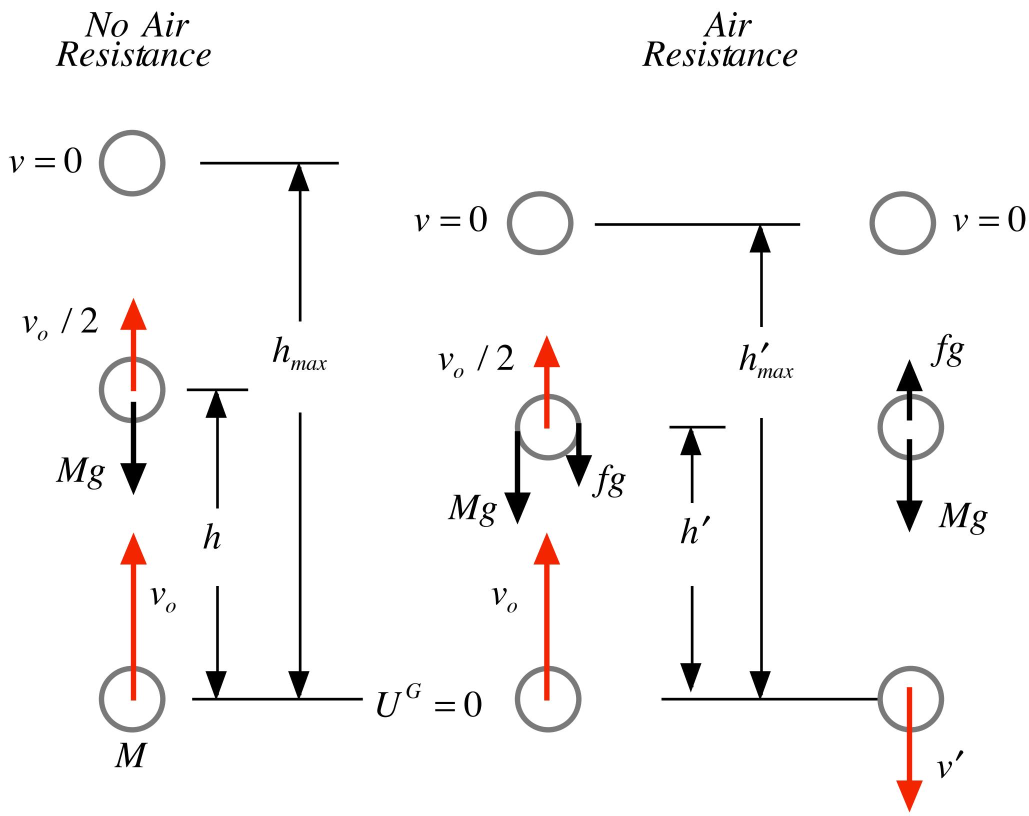
$$\Delta K = -\frac{1}{2}\Delta U^G\ , \tag{9}$$

and not

$$\Delta K = -\Delta U^G\ , \tag{10}$$

as we might have expected from the conservation of mechanical energy. Why is the mechanical energy not conserved? It turns out that we can not treat these objects, the Sun and the Earth, as point masses with no internal structure. The relationship expressed by equation (9) is know as the virial theorem and is valid for all central forces. You will see next semester that this relationship also holds for electrons in "circular" orbits of their respective nucleus. (Others, Fritz Zwicky for example, have argued that this is evidence that there is "background matter" that is not visible--so-called dark matter.)

6.) Solution:



a) If there is no air resistance, the only force doing work would be the conservative gravitational force and we could use the conservation of mechanical energy. So, we write

$$K_o + U_o^G = K_L + U_L^G\ , \tag{1}$$

$$\frac{1}{2}Mv_o^2 + 0 = \frac{1}{2}Mv^2 + Mgh \ , \tag{2}$$

solving for h we find

$$h = \frac{v_o^2 - v^2}{2g} \ . \tag{3}$$

When the stone has half of its initial speed, its distance above the ground is given by

$$h = \frac{v_o^2 - \left(v_o / 2\right)^2}{2g} = \frac{3}{8}\frac{v_o^2}{g} = \frac{3}{8}\frac{\left(17.882 \ m/s\right)^2}{\left(9.805 \ m/s^2\right)} = 12.23 \ m . \tag{4}$$

b) The maximum height is given by

$$h_{max} = \frac{v_o^2 - \left(0^2\right)}{2g} = \frac{v_o^2}{2g} = \frac{\left(17.882 \ m/s\right)^2}{2\left(9.805 \ m/s^2\right)} = 16.31 \ m \ . \tag{5}$$

c) If there is air resistance, we would then have a non-conservative frictional force that we would have to include.

(a) We write

$$W_f + W_G = \Delta K = K_L - K_o \ . \tag{6}$$

$$W_f = (\mu g)(h')(cos\angle_{bet}) = (\mu g)(h')(cos 180°) = -\mu g h' \ . \tag{7}$$

$$W_G = -\Delta U^G = U_o^G - U_L^G = 0 - Mgh' \ . \tag{8}$$

Substitution of equations (7) and (8) into (6) gives us

$$-\mu g h' - Mgh' = \frac{1}{2}M\left[v_o / 2\right]^2 - \frac{1}{2}Mv_o^2 = -\frac{3}{8}Mv_o^2 \ . \tag{9}$$

Solving for h' we have

$$h' = \frac{-(3/8)Mv_o^2}{-[\mu + M]g} = \left[\frac{M}{\mu + M}\right]\left[\frac{3}{8}\frac{v_o^3}{g}\right] = \left[\frac{M}{\mu + M}\right]h$$

$$= \left[\frac{0.5 \ kg}{0.0125 + 0.5 \ kg}\right]\left[12.23 \ m\right] = 11.93 \ m \ . \tag{10}$$

Note that if $\mu = 0$, we would have our result from a).

(b) We can modify equation (9) to match the constraints. We have

$$-\mu g h'_{max} - Mgh'_{max} = 0 - \frac{1}{2}Mv_o^2 = -\frac{1}{2}Mv_o^2 \ , \tag{11}$$

and

$$h'_{max} = \frac{-(1/2)Mv_o^2}{-(\mu + M)g} = \left[\frac{M}{\mu + M}\right]\left[\frac{v_o^2}{2g}\right] = \left[\frac{M}{\mu + M}\right]h_{max}$$

$$= \left[\frac{0.5 \ kg}{0.0125 \ kg + 0.5 \ kg}\right]16.31 \ m = 15.91 \ m \ . \tag{12}$$

d) On the downward phase, the frictional and gravitational forces are in opposite directions. So, we have

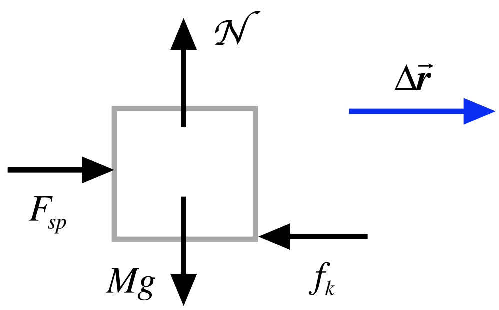
$$-\mu g h'_{max} + M g h'_{max} = \frac{1}{2} M v'^2 \ , \tag{13}$$

and

$$v' = \sqrt{\frac{2(M-\mu)}{M} g h'_{max}} = \sqrt{2\left(1-\frac{\mu}{M}\right) g h'_{max}}$$

$$= \sqrt{2\left(1-\frac{0.0125\ kg}{0.5\ kg}\right)(9.805\ m/s^2)(15.91\ m)} = 17.44\ m/s \ . \tag{14}$$

7.) Solution:



We can write

$$W_{\mathcal{N}} + W_G + W_{f_k} + W_{sp} = \Delta K \ . \tag{1}$$

Note that both the normal force and the gravitational force are everywhere perpendicular to the displacement and so do no work on the block, i.e.

$$W_{\mathcal{N}} = W_G = 0 \ . \tag{2}$$

The work done on the block by the kinetic frictional force is given by

$$W_{f_k} = (f_k)(\Delta r)(cos \angle_{bet}) = (\mu_k M g)(\Delta r)(cos 180°) = -\mu_k M g (\Delta r) \ . \tag{3}$$

The work done on the block by the elastic spring is given by

$$W_{sp} = U_o^{sp} - U_L^{sp} = \frac{1}{2} k_{sp} x_o^2 - \frac{1}{2} k_{sp} x_L^2 = \frac{1}{2} k_{sp} \left[x_o^2 - x_L^2\right] . \tag{4}$$

The system is released from rest so the initial kinetic energy is zero. We can substitute equations (2), (3) and (4) into equation (1) getting

$$0 + 0 - \mu_k M g (\Delta r) + \frac{1}{2} k_{sp} \left[x_o^2 - x_L^2\right] = \frac{1}{2} M v^2 \ . \tag{5}$$

Solving for v we find

$$v = \sqrt{\frac{2}{M}\left\{-\mu_k M g(\Delta r) + \frac{1}{2} k_{sp}\left[x_o^2 - x_L^2\right]\right\}}$$

$$= \sqrt{\frac{k_{sp}}{M}\left[x_o^2 - x_L^2\right] - 2\mu_k g(\Delta r)} \ . \tag{6}$$

a) When the block is released, the spring has been compressed twelve *centimeters*. The spring will accelerate the block and at the point the spring is compressed only six *centimeters*, half of its original compression, the block will have some speed v' given by

$$v' = \sqrt{\frac{k_{sp}}{M}\left[x_o^2 - \left(\frac{1}{2}x_o\right)^2\right] - 2\mu_k g\left(\frac{1}{2}x_o\right)} = \sqrt{\frac{3}{4}\frac{k_{sp}}{M}x_o^2 - \mu_k g x_o}$$

$$= \sqrt{\frac{3}{4}\frac{(5125\ N/m)}{(1.25\ kg)}(0.12\ m)^2 - (0.68)\left(9.805\ \frac{m}{s^2}\right)(0.12\ m)} = 6.594\ \frac{m}{s}\ . \quad (7)$$

b) At that point where the spring is no longer compressed, the block will have a speed v_{nc} given by

$$v_{nc} = \sqrt{\frac{k_{sp}}{M}\left[x_o^2 - (0)^2\right] - 2\mu_k g(x_o)} = \sqrt{\frac{k_{sp}}{M}x_o^2 - 2\mu_k g x_o}$$

$$= \sqrt{\frac{(5{,}125\ N/m)}{(1.25\ kg)}(0.12\ m)^2 - 2(0.68)\left(9.805\ \frac{m}{s^2}\right)(0.12\ m)} = 7.579\frac{m}{s}\ . \quad (8)$$

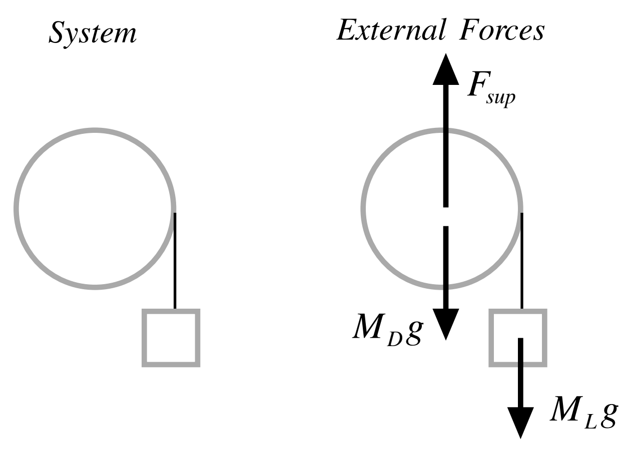
c) Once the block is no longer in contact with the spring, its speed decreases because of friction. Eventually, the block stops some distance ℓ from the starting point. So, using equation (6) we can write

$$0^2 = \frac{k_{sp}}{M}\left[x_o^2 - (0)^2\right] - 2\mu_k g(\ell)\ , \quad (9)$$

and

$$\ell = \frac{k_{sp}x_o^2}{2\mu_k Mg} = \frac{(5{,}125\ N/m)(0.12\ m)^2}{2(0.68)(1.25\ kg)(9.805\ m/s^2)} = 4.428\ m\ . \quad (10)$$

8.) Solution:



We take the disk, the block and the cable as the system of interest. The only external forces of are the support force exerted on the disk by the axle, and the gravitational forces exerted on the disk and block by the Earth. As the support force does not displace anything, it does no work. So, the only force doing work is the conservative gravitational force. We use, then, the conservation of mechanical energy. We write

$$K_o + U_o^G = K_L + U_L^G \ . \tag{1}$$

Since the system is released from rest, there is no initial kinetic energy. Also, we assign the later gravitational potential energy to be zero. Equation (1) becomes

$$0 + M_L g\ell = \frac{1}{2}M_L v_L^2 + \frac{1}{2}I_D\omega_L^2 + 0 \ . \tag{2}$$

The moment of inertia of the disk is

$$I_D = \frac{1}{2}M_D R^2 \ . \tag{3}$$

Since the cable is not slipping, the angular speed is related to the linear speed by

$$v_L = R\omega_L \, . \tag{4}$$

Substitution of equations (3) and (4) into equation (2) gives us

$$M_L g\ell = \frac{1}{2}M_L v_L^2 + \frac{1}{2}\left(\frac{1}{2}M_D R^2\right)\left(\frac{v_L}{R}\right)^2 = \frac{1}{2}\left[M_L + \frac{1}{2}M_D\right]v_L^2 \ , \tag{5}$$

and

$$v_L = \sqrt{\left[\frac{M_L}{M_L + (1/2)M_D}\right]2g\ell}$$

$$= \sqrt{\left[\frac{125\ kg}{125\ kg + (75/2)\ kg}\right]2\left(9.805\ m/s^2\right)\left(1.25\ m\right)} = 4.342\ \frac{m}{s} \ . \tag{6}$$

b) For constant acceleration with an initial speed of zero, we know

$$v_L = \sqrt{2|a|\ell} \ . \tag{7}$$

Comparing equations (6) and (7) should convince one that

$$|a| = \left[\frac{M_L}{M_L + (1/2)M_D}\right]g$$

$$= \left[\frac{125\ kg}{125\ kg + (75/2)\ kg}\right]\left(9.805\ m/s^2\right) = 7.542\ \frac{m}{s^2} \ . \tag{8}$$

9.) Solution:

We can write

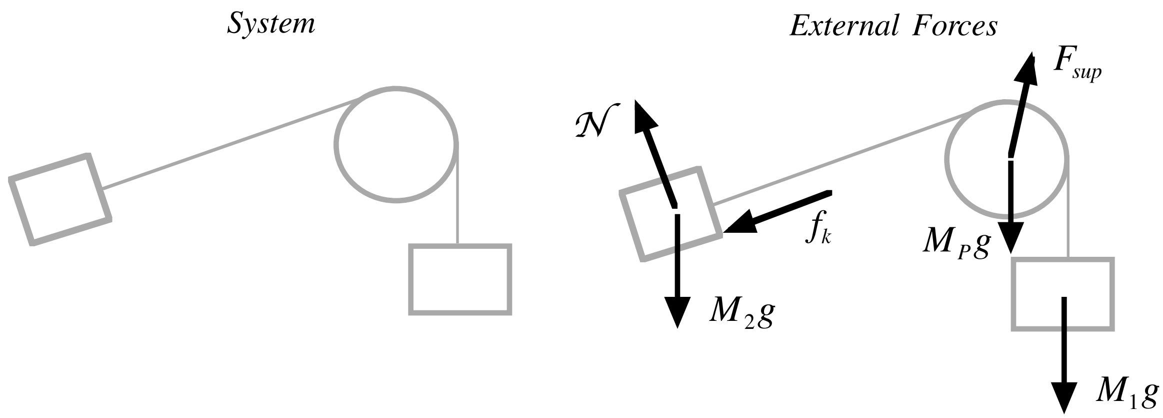
$$W_{F_{sup}} + W_{\mathcal{N}} + W_{f_k} + W_G = K_L - K_o \ . \tag{1}$$

As the support force does not displace any physical thing, it does no work. As the normal force is everywhere perpendicular to the path of block two, it does no work. As the system is released from rest, the initial kinetic energy is zero. The work done by the kinetic frictional force is given by

$$W_{f_k} = (f_k)(\Delta r)(cos\angle_{bet}) = (\mu_k M_2 g\cos\theta)(\ell')(cos180^\circ) = -\mu_k M_2 g\ell'\cos\theta \ . \tag{2}$$

The work done by the gravitational force is given by

$$W_G = U_o^G - U_L^G = U_o^G - 0 = M_1 g\ell' - M_2 g\ell' \sin\theta \ . \tag{3}$$



Later, the blocks have translational kinetic energy and the pulley has rotational kinetic energy. So, we can write

$$K_L = \frac{1}{2}\left(M_1 + M_2\right)v_L^2 + \frac{1}{2}I\omega_L^2 = \frac{1}{2}\left(M_1 + M_2\right)v_L^2 + \frac{1}{2}\left(\frac{1}{2}M_P R^2\right)\left(\frac{v_L}{R}\right)^2$$

$$= \frac{1}{2}\left(M_1 + M_2 + \left(1/2\right)M_P\right)v_L^2 \ . \qquad (4)$$

We can now rewrite equation (1) as

$$0 + 0 - \mu_k M_2 g\,\ell' \cos\theta + M_1 g\,\ell' - M_2 g\,\ell' \sin\theta = \frac{1}{2}\left(M_1 + M_2 + \left(1/2\right)M_P\right)v_L^2 - 0 \ .(5)$$

So,

$$v_L = \sqrt{\left[\frac{M_1 - M_2\left(\sin\theta + \mu_k \cos\theta\right)}{M_1 + M_2 + \left(1/2\right)M_P}\right]2g\ell'}$$

$$= \sqrt{\left[\frac{\left(8 - 4\left(\sin 25° + (.25)\cos 25°\right)\right)kg}{\left(8 + 4 + (1/2)2\right)kg}\right]2\left(9.805\ m/s^2\right)\left(5.75\ m\right)} = 6.846\,\frac{m}{s} \ .(6)$$

b) For constant acceleration with an initial speed of zero, we know

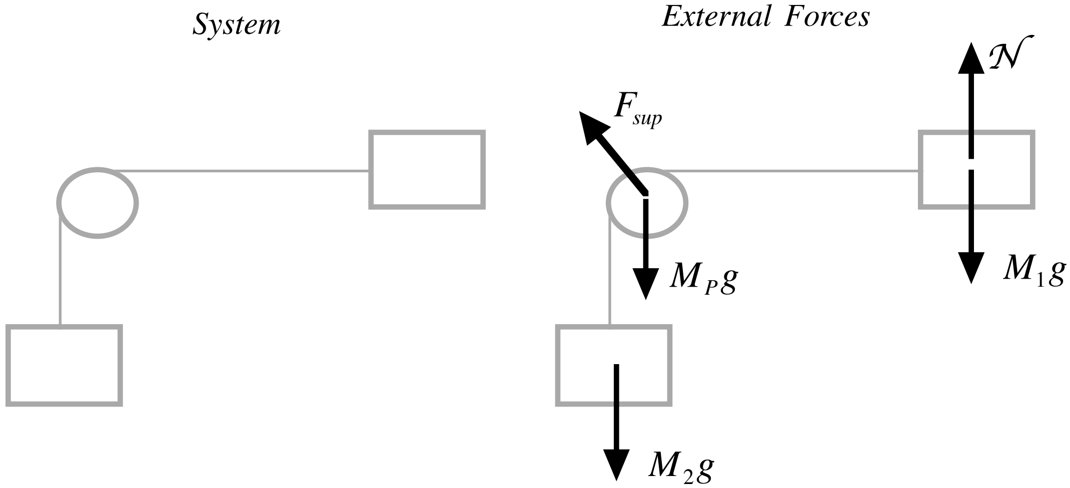
$$v_L = \sqrt{2\left|a\right|\ell} \ . \qquad (7)$$

Comparing equations (6) and (7) should convince one that

$$\left|a\right| = \left[\frac{M_1 - M_2\left(\sin\theta + \mu_k \cos\theta\right)}{M_1 + M_2 + \left(1/2\right)M_P}\right]g$$

$$= \left[\frac{\left(8 - 4\left(\sin 25° + (.25)\cos 25°\right)\right)kg}{\left(8 + 4 + (1/2)2\right)kg}\right]\left(9.805\ m/s^2\right) = 4.075\,\frac{m}{s^2} \ . \qquad (8)$$

10.) Solution:



We can write

$$W_{F_{sup}} + W_{\mathcal{N}} + W_G = K_L - K_o \ . \qquad (1)$$

As the support force does not displace any physical thing, it does no work. Also, the normal force is everywhere perpendicular to the path of block one, it does no work. The system is released from rest so, the initial kinetic energy is zero. The work done by the gravitational force is given by

$$W_G = U_o^G - U_L^G = U_o^G - 0 = M_2 g\ell \ . \qquad (2)$$

Later, the blocks have translational kinetic energy and the pulley has rotational kinetic energy. So, we can write

$$K_L = \frac{1}{2}(M_1 + M_2)v_L^2 + \frac{1}{2}I\omega_L^2 = \frac{1}{2}(M_1 + M_2)v_L^2 + \frac{1}{2}\left(\frac{1}{2}M_P R^2\right)\left(\frac{v_L}{R}\right)^2$$

$$= (1/2)\big(M_1 + M_2 + (1/2)M_P\big)v_L^2 \ . \qquad (3)$$

We can now rewrite equation (1) as

$$0 + 0 + M_2 g\ell = \frac{1}{2}\big(M_1 + M_2 + (1/2)M_P\big)v_L^2 - 0 \ . \qquad (4)$$

So,

$$v_L = \sqrt{\left[\frac{M_2}{M_1 + M_2 + (1/2)M_P}\right]2g\ell}$$

$$= \sqrt{\left[\frac{8\ kg}{\big(2 + 8 + (1/2)(1/2)\big)kg}\right]2\left(9.805\ m/s^2\right)(.75\ m)} = 3.39\frac{m}{s} \ . \qquad (5)$$

b) For constant acceleration with an initial speed of zero, we know

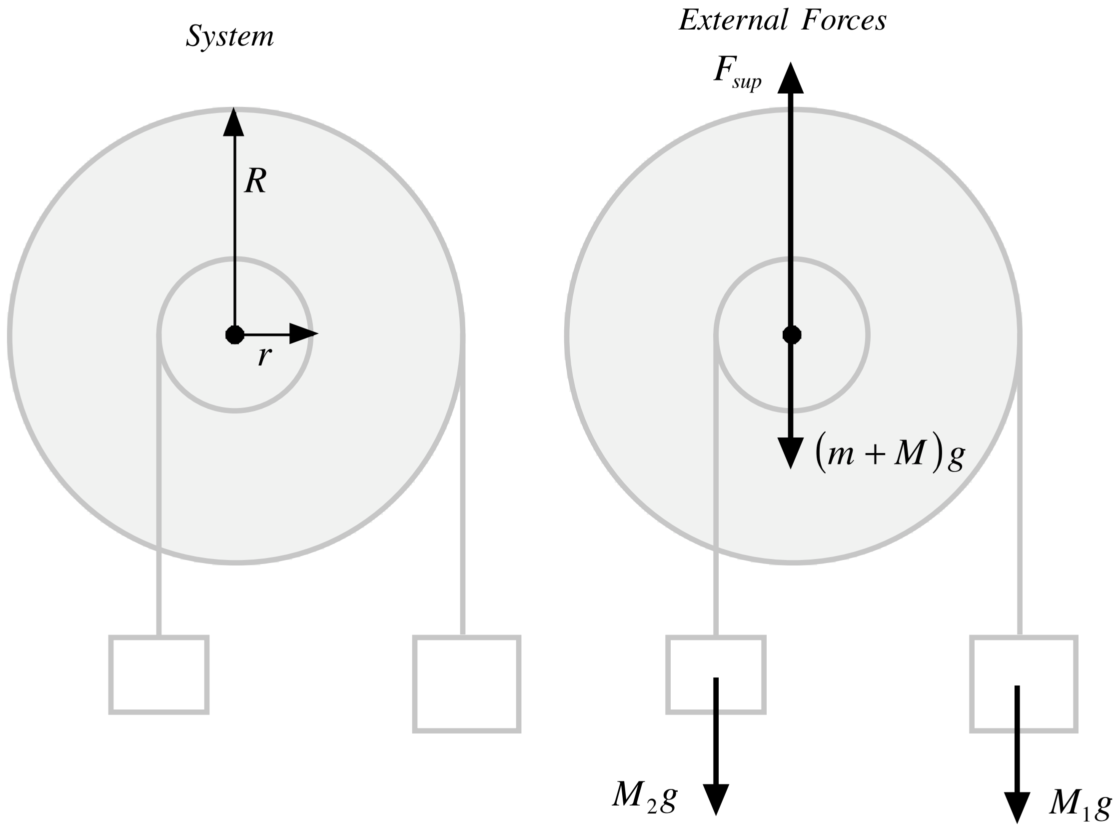
$$v_L = \sqrt{2|a|\ell} \ . \qquad (6)$$

Comparing equations (5) and (6) should convince one that

$$|a| = \left[\frac{M_2}{M_1 + M_2 + (1/2)M_P}\right]g$$

$$|a| = \left[\frac{8\ kg}{2\ kg + 8\ kg + (1/2)((1/2)\ kg)} \right] (9.805\ m/s^2) = 7.65\ \frac{m}{s^2}\ . \qquad (7)$$

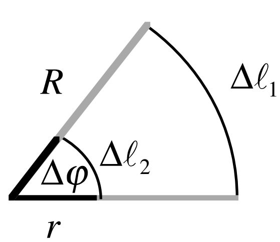
11.) Solution:



a) The support force does not displace any physical thing so it does no work. The only force doing work is the conservative gravitational force. So, we can use the conservation of mechanical energy. We write

$$K_o + U_o^G = K_L + U_L^G\ . \qquad (1)$$

The major complicating factor of this problem is that the blocks do not move at the same speeds. The diagram below indicates how to deal with this.



W can see that in a time interval Δt, a point on the rim of the large cylinder moves through a distance $\Delta\ell_1$, while a point on the rim of the small cylinder moves through a distance $\Delta\ell_2$. As these paths are circular arcs, we can write

$$\Delta\ell_1 = R\ \Delta\varphi\ , \tag{2}$$

and
$$\Delta\ell_2 = r\ \Delta\varphi \tag{3}$$

Substitution of equation (3) into equation (2) gives us

$$\Delta\ell_1 = R\ \Delta\varphi = R\frac{\Delta\ell_2}{r} = \frac{R}{r}\Delta\ell_2 = 3\ \Delta\ell_2\ . \tag{4}$$

So, we conclude that since block one moves three times farther than block two in the same time interval, then

$$v_1 = 3v_2 \ \text{ and } \ a_1 = 3a_2\ . \tag{5}$$

Also, then

$$\omega_L = \frac{v_1}{R} = \frac{v_2}{r} = \frac{(1/3)v_1}{(1/3)R}\ . \tag{6}$$

Now, we can solve equation (1).

Since the system is released from rest, the initial kinetic energy is zero. Also, we arbitrarily assign the later gravitational potential energy to be zero. So, we can rewrite equation (1) as

$$0 + M_1 g\ell - M_2 g(\ell/3) = \frac{1}{2}M_1 v_{L1}^2 + \frac{1}{2}M_2 v_{L2}^2 + \frac{1}{2}I_M\omega_L^2 + \frac{1}{2}I_m\omega_L^2 + 0\ , \tag{7}$$

Note:
$$I_m = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{1}{9}M\right)\left(\frac{1}{3}R\right)^2 = \left(\frac{1}{162}MR^2\right), \tag{8}$$

and
$$I_M = \frac{1}{2}MR^2 = \frac{81}{162}MR^2\ . \tag{9}$$

Substitution of the results of equations (2) through (6) and (8) and (9) into equation (2) gives us

$$\left(M_1 - \frac{M_2}{3}\right)g\ell = \frac{1}{2}\left[M_1 v_{L1}^2 + M_2\left(\frac{v_{L1}}{3}\right)^2 + \left[\left(\frac{81}{162}MR^2\right) + \left(\frac{1}{162}MR^2\right)\right]\left(\frac{v_{L1}}{R}\right)^2\right], \tag{10}$$

$$\left[M_1 - (M_2/3)\right]2g\ell = \left[M_1 + (1/9)M_2 + (41/81)M\right]v_{L1}^2\ , \tag{11}$$

$$v_{L1} = \sqrt{\left[\frac{M_1 - (M_2/3)}{M_1 + (1/9)M_2 + (41/81)M}\right]2g\ell}$$

$$= \sqrt{\left[\frac{(18 - (6/3))\,kg}{(18 + (1/9)(6) + (41/81)(18))\,kg}\right]2(9.805\ m/s^2)(.75\ m)} = 2.911\frac{m}{s}. \tag{12}$$

Also, then,
$$v_{L2} = (1/3)v_{L1} = (1/3)(2.911\ m/s) = 0.970\ m/s\ . \tag{13}$$

b) Equation (12) implies that the magnitude of the acceleration of block one is given by

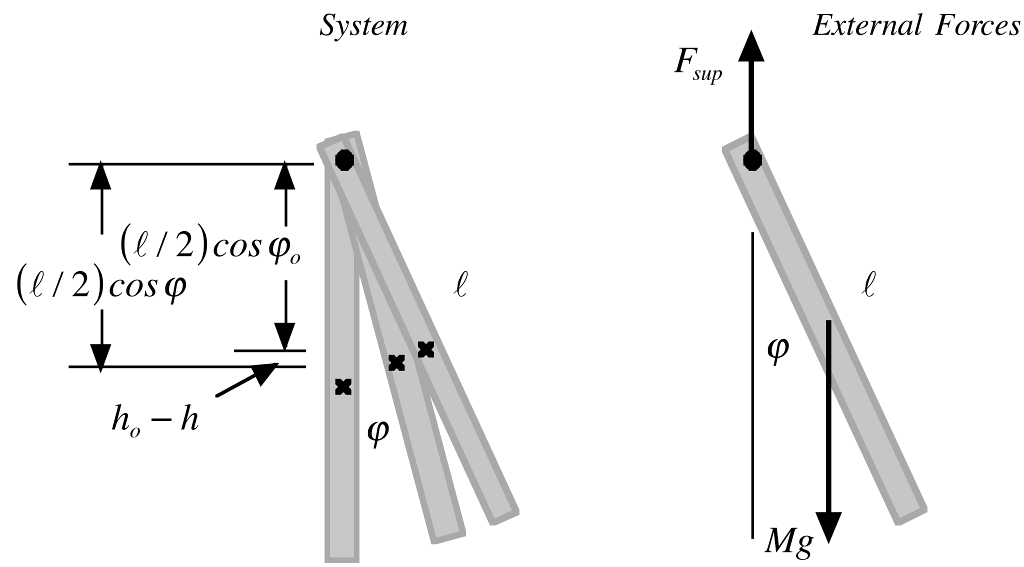
$$|a_1| = \left[\frac{M_1 - (M_2/3)}{M_1 + (1/9)M_2 + (41/81)M}\right]g$$

$$= \left[\frac{(18 - (6/3))\,kg}{(18 + (1/9)(6) + (41/81)(18))\,kg}\right](9.805\ m/s^2) = 5.648\frac{m}{s^2}\ . \tag{14}$$

Therefore,

$$|a_2| = (1/3)|a_1| = (1/3)(5.648\ m/s^2) = 1.883\ m/s^2\ . \tag{15}$$

12.) Solution:



a) Since the support force does not displace any physical thing, it does no work. So, we can use the conservation of mechanical energy. We write

$$K_o + U_o^G = K_L + U_L^G\ . \tag{1}$$

As the system is released from rest, the initial rotational kinetic energy is zero. We assign the gravitational potential energy to be zero at that vertical height at which we find the center of mass as the rod passes through the vertical. So, we can rewrite equation (1) as

$$0 + Mgh_o = (1/2)I\omega^2 + Mgh\ . \tag{2}$$

So, in general,

$$\omega = \sqrt{\frac{2Mg}{I}(h_o - h)}\ . \tag{3}$$

Note:

$$I = (1/3)M\ell^2\ . \tag{4}$$

Also,

$$h_o - h = \left[\frac{\ell}{2}\cos\varphi - \frac{\ell}{2}\cos\varphi_o\right] = \frac{\ell}{2}(\cos\varphi - \cos\varphi_o)\ . \tag{5}$$

Substitution of equations (4) and (5) into equation (3) gives us

$$\omega = \sqrt{\frac{2Mg}{I}(h_o - h)} = \sqrt{\frac{2Mg}{(1/3)M\ell^2}\frac{\ell}{2}(\cos\varphi - \cos\varphi_o)}$$

$$= \sqrt{\frac{3g}{\ell}(\cos\varphi - \cos\varphi_o)}\ . \tag{6}$$

When the rod passes through the vertical, $\varphi = 0$ and the angular speed is

$$\omega_{max} = \sqrt{\frac{3\left(9.805\ m/s^2\right)}{\left(2.25\ m\right)}\left(cos\,0° - cos\,25°\right)} = 1.107\,\frac{rad}{s}\ . \qquad (7)$$

b) Here we wish to find an angle φ knowing an angular speed. So, we solve equation (6) for φ. We find

$$\omega^2 = \frac{3g}{\ell}\left(cos\,\varphi - cos\,\varphi_o\right)\ , \qquad (8)$$

and

$$cos\,\varphi - cos\,\varphi_o = \frac{\omega^2 \ell}{3g}\ , \qquad (9)$$

so

$$cos\,\varphi = \frac{\omega^2 \ell}{3g} + cos\,\varphi_o\ . \qquad (10)$$

Therefore,

$$\varphi = cos^{-1}\left[\frac{\omega^2 \ell}{3g} + cos\varphi_o\right]\ . \qquad (11)$$

Note, if C is a positive constant such that $0 \le C \le 1$, then we can write

$$\omega = C\omega_{max} = C\sqrt{\frac{3g}{\ell}\left[1 - cos\,\varphi_o\right]}\ , \qquad (12)$$

and, of course,

$$\omega^2 = \left(C\omega_{max}\right)^2 = \frac{3gC^2}{\ell}\left[1 - cos\,\varphi_o\right]\ . \qquad (13)$$

Substitution of equation (13) into equation (11) gives us

$$\varphi = cos^{-1}\left[\frac{\left[\left(3gC^2/\ell\right)\left[1 - cos\,\varphi_o\right]\right]\ell}{3g} + cos\,\varphi_o\right]$$

$$= cos^{-1}\left[C^2\left(1 - cos\varphi_o\right) + cos\varphi_o\right]\ . \qquad (14)$$

To find the angle at which we have half of the maximum speed, the $C = 1/2$, and we have

$$\varphi = cos^{-1}\left[\frac{1}{4}\left(1 - cos\,25°\right) + cos\,25°\right] = 21.6°\ . \qquad (15)$$

Such is the sick, twisted way a physicist might think. Most students would, of course, simply use equations (7) and (11) getting

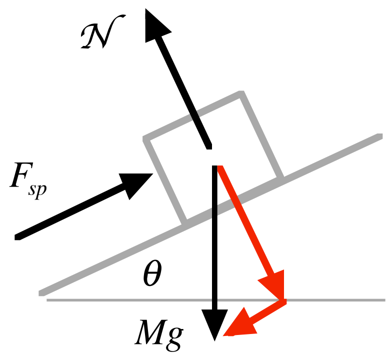
$$\varphi = cos^{-1}\left[\frac{\omega^2 \ell}{3g} + cos\varphi_o\right]$$

$$= cos^{-1}\left[\frac{\left(\left(1.107\ rad/s\right)/2\right)^2\left(2.25\ m\right)}{3\left(9.805\ m/s^2\right)} + cos\,25°\right] = 21.6°\ . \qquad (16)$$

Of course, the major point of this whole exercise is to show you through how little angle the rod swings before it already has one half of the speed it will have at the bottom.

13.) Solution:

External Forces



From the time the block is released until it makes contact with the spring, it will be acted on by a normal force and the conservative gravitational force. When it is also in contact with the spring then the spring will exert a conservative force. As the normal force is everywhere perpendicular to the path of the block, it will do no work on the block. Since only conservative forces will do work on the block, we can use the conservation of mechanical energy. We write

$$K_o + U_o^G + U_o^{sp} = K_L + U_L^G + U_L^{sp} \ . \tag{1}$$

a) Over the interval of release to initial contact with the spring, the block does not make contact with the spring so it will do no work. For this interval, equation (1) is

$$0 + Mg\ell \, sin\theta + 0 = \frac{1}{2} M v_{max}^2 + 0 + 0 \ , \tag{2}$$

where we have set $U_L^G = 0$. So, equation (2) implies

$$v_{max} = \sqrt{2g\ell \, sin\theta} = \sqrt{2\left(9.805 \ m/s^2\right)\left(2.25 \ m\right)\left(sin 25°\right)} = 4.318 \ m/s \, . \tag{3}$$

b) Rewriting equation (1) for the interval of initial contact to one half maximum speed, we have

$$\frac{1}{2} M \left[\sqrt{2g\ell \, sin\theta}\right]^2 + Mgx \, sin\theta + 0 = \frac{1}{2} M \frac{1}{2}\left[\sqrt{2g\ell \, sin\theta}\right]^2 + 0 + \frac{1}{2} k_{sp} x^2 \ , \tag{4}$$

where, again, $U_L^G = 0$. We rearrange the equation into quadratic form. We have

$$\frac{1}{2} k_{sp} x^2 - \left(Mg \, sin\theta\right) x - \frac{3}{4} \ell \left(Mg \, sin\theta\right) = 0 \ . \tag{5}$$

Multiplying each term by two and dividing each term by the spring constant gives us

$$x^2 - 2\left(\frac{Mg \, sin\theta}{k_{sp}}\right) x - \frac{3}{2} \ell \left(\frac{Mg \, sin\theta}{k_{sp}}\right) = 0 \ . \tag{6}$$

Note:

$$\left(\frac{Mg \, sin\theta}{k_{sp}}\right) = \frac{\left(6.750 \ kg\right)\left(9.805 \ m/s^2\right)\left(sin 25°\right)}{\left(4{,}750 \ N/m\right)} = 0.005889 \ m \ , \tag{7}$$

So,

$$x = \left(0.005889 \ m\right) + \sqrt{\left(0.005889 \ m\right)^2 + \left(3.375 \ m\right)\left(0.005889 \ m\right)} = 0.147 \ m \ . \tag{8}$$

c) To find the maximum compression of the spring, x_c, it is helpful to use the entire interval from start to finish. We write, using equation (1)

$$0 + Mg\left(\ell + x_c\right)sin\theta + 0 = 0 + 0 + \frac{1}{2}k_{sp}x_c^2 \ . \tag{9}$$

So,

$$\frac{1}{2}k_{sp}x_c^2 - \left(Mg\,sin\,\theta\right)x_c - Mg\ell\,sin\,\theta = 0 \ , \tag{10}$$

and

$$x_c^2 - 2\left(\frac{Mg\ sin\theta}{k_{sp}}\right)x_c - 2\ell\left(\frac{Mg\,sin\,\theta}{k_{sp}}\right) = 0 \ . \tag{11}$$

Finally,

$$x_c = \left(\frac{Mg\,sin\,\theta}{k_{sp}}\right) + \sqrt{\left(\frac{Mg\,sin\,\theta}{k_{sp}}\right)^2 + 2\ell\left(\frac{Mg\,sin\,\theta}{k_{sp}}\right)}$$

$$= \left(0.005889\ m\right) + \sqrt{\left(0.005889\ m\right)^2 + \left(4.50\ m\right)\left(0.005889\ m\right)} = 0.169\ m \ . \tag{12}$$

14.) Solution:

If we ignore air resistance, then the conservative gravitational force is the only force doing work on the ball. Again, we use the conservation of mechanical energy. We can write

$$K_o + U_o^G = K_L + U_L^G \ , \tag{1}$$

$$\frac{1}{2}Mv_o^2 + Mgh_o = \frac{1}{2}Mv^2 + Mgh \ , \tag{2}$$

where we have set the gravitational potential energy to be zero at sea level. Multiplying by two and dividing by the mass we have

$$v_o^2 + 2gh_o = v^2 + 2gh. \tag{3}$$

a) The maximum speed is reached just as the ball strikes the ocean; where $h = 0$. So, using equation (3) we have

$$v_{max} = \sqrt{v_o^2 + 2gh_o} = \sqrt{\left(18\ m/s\right)^2 + 2\left(9.805\ m/s^2\right)\left(20\ m\right)} = 26.8\ m/s \ . \tag{4}$$

b) In general, using equation (3), we can write

$$h = h_o + \frac{v_o^2 - v^2}{2g}$$

$$= \left(20\ m\right) + \frac{\left(18\ m/s\right)^2 - \left[\left(3/4\right)26.8\ m/s\right]^2}{2\left(9.805\ m/s^2\right)} = 15.9\ m \ . \tag{5}$$

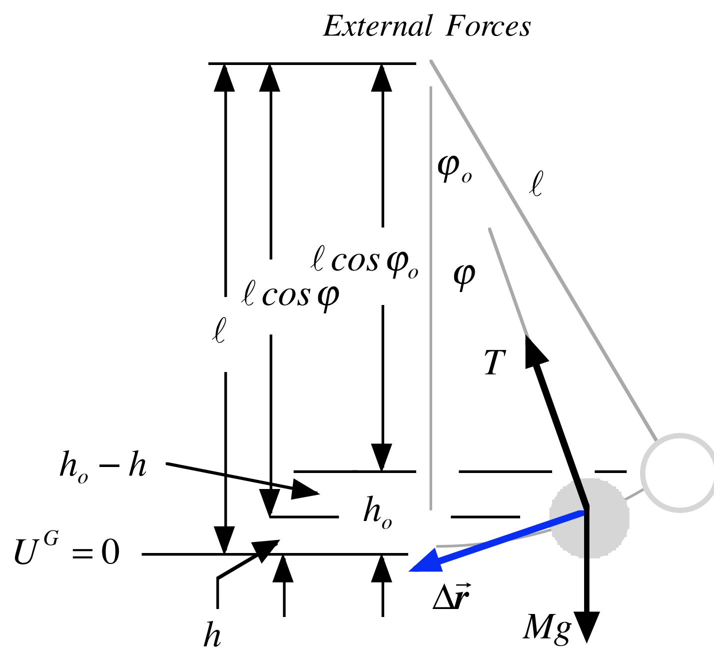
15.) Solution:

Since the tension is always perpendicular to the path of motion of the bob, it does no work. We can, then, use the conservation of mechanical energy because the only force doing work is the conservative gravitational force. The initial kinetic energy is zero, and we define the gravitational potential energy to be zero when the bob is a the lowest point in its path. We can write

$$Mgh_o = (1/2)Mv^2 + Mgh \ . \tag{1}$$

and

$$v = \sqrt{2g(h_o - h)} \ . \tag{2}$$



Note:

$$h_o - h = \ell \cos\varphi - \ell \cos\varphi_o = \ell(\cos\varphi - \cos\varphi_o) \ . \tag{3}$$

Therefore,

$$v = \sqrt{2g\ell(\cos\varphi - \cos\varphi_o)} \ . \tag{4}$$

a) When the bob passes through the vertical, $\varphi = 0°$ and $\cos 0° = 1$. So, the maximum speed of the bob is given by

$$v_{max} = \sqrt{2g\ell(1 - \cos\varphi_o)}$$

$$= \sqrt{2(9.805 \ m/s^2)(1.675 \ m)(1 - \cos 30°)} = 2.098 \ m/s \ . \tag{5}$$

b) To find the angle at which the bob has a certain speed, we rearrange equation (5).

$$v^2 = 2g\ell(\cos\varphi - \cos\varphi_o) \ , \tag{6}$$

$$\cos\varphi - \cos\varphi_o = \frac{v^2}{2g\ell} \ , \tag{7}$$

$$\varphi = \cos^{-1}\left[\frac{v^2}{2g\ell} + \cos\varphi_o\right] . \tag{8}$$

Now, if we assume that

$$v = Cv_{max} = C\sqrt{2g\ell\left(1 - cos\varphi_o\right)} \ , \tag{9}$$

where C is a constant such that $0 \le C \le 1$, then we can rewrite equation (8) as

$$\varphi = cos^{-1}\left[\frac{C^2\left(2g\ell\left(1 - cos\,\varphi_o\right)\right)}{2g\ell} + cos\,\varphi_o\right]$$

$$= cos^{-1}\left[C^2\left(1 - cos\varphi_o\right) + cos\varphi_o\right] \ . \tag{10}$$

i) $C = 1/4$ and

$$\varphi = cos^{-1}\left[(1/4)^2\left(1 - cos\,30°\right) + \left(cos\,30°\right)\right] = cos^{-1}\left[0.87440\right] = 29.0° \ . \tag{11}$$

ii) $C = 1/2$ and

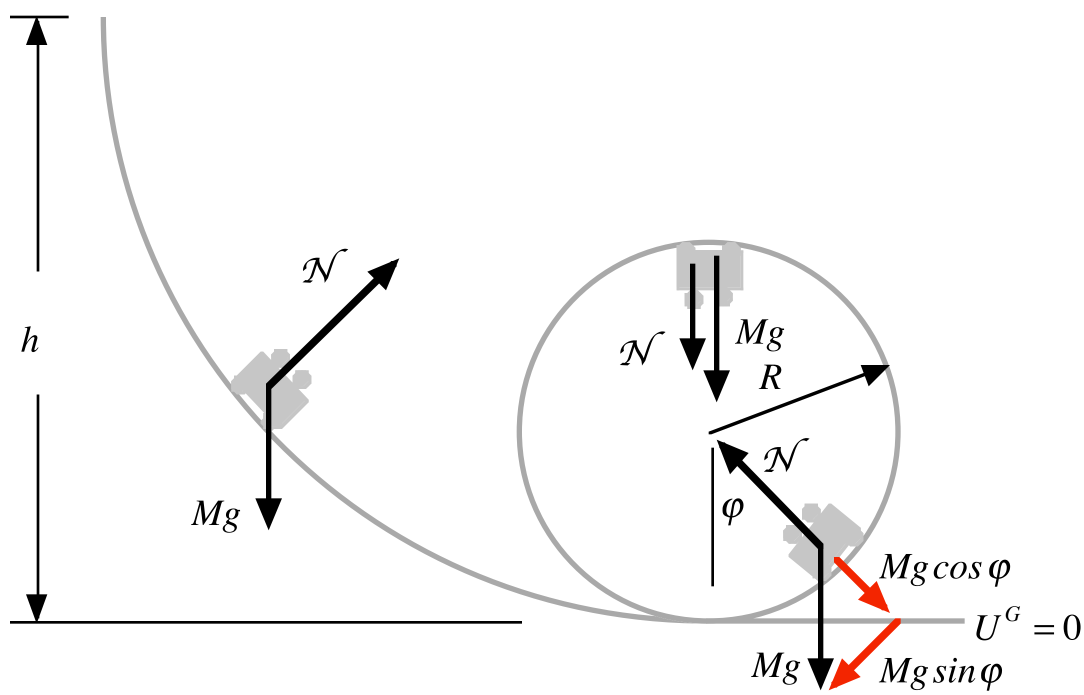
$$\varphi = cos^{-1}\left[(1/2)^2\left(1 - cos\,30°\right) + \left(cos\,30°\right)\right] = cos^{-1}\left[0.89952\right] = 25.9° \ . \tag{12}$$

i) $C = 3/4$ and

$$\varphi = cos^{-1}\left[(3/4)^2\left(1 - cos\,30°\right) + \left(cos\,30°\right)\right] = cos^{-1}\left[0.94138\right] = 19.7° \ . \tag{13}$$

16.) Solution:

External Forces



As the normal force is perpendicular to the path at each point on the path, it does no work on the car. We can use the conservation of mechanical energy as the conservative gravitational force is the only force doing work on the car. We assign the gravitational potential energy to be zero when the car is at the lowest point of the loop. So, we can write

$$K_o + U_o^G = K_L + U_L^G \ . \tag{1}$$

a) First, we calculate the speed of the car at the bottom of the hill as it enters the loop-the-loop. For this interval, we can write

$$0 + Mgh = \frac{1}{2}Mv_{bot}^2 + 0 \ , \tag{2}$$

and, therefore,
$$v_{bot} = \sqrt{2gh} \ . \tag{3}$$

Next, we consider the requirements at the top of the loop-the-loop of minimum speed without "falling off" the track. As the path is circular, we know that

$$-\mathcal{N} - Mg = -\frac{Mv_{top}^2}{R} \ , \tag{4}$$

and, therefore,

$$v_{top} = \sqrt{\frac{R}{M}\left(\mathcal{N} + Mg\right)} \ . \tag{5}$$

The normal force represents the force exerted on the car by the track. If the car were to "fall off," then at the instant it lost contact with the track the normal force would become zero! So, zero represents the smallest possible value for the normal and it must not happen before the car gets to the top. So, setting the normal to zero in equation (5) will give us the minimum speed at the top required for the car to "stay on track." We have

$$v_{top,min} = \sqrt{Rg} \ . \tag{6}$$

Now, we use equation (1) over the interval where the car moves from the bottom to the top of the loop-the-loop. We find

$$\frac{1}{2}M\left(\sqrt{2gh}\right)^2 + 0 = \frac{1}{2}M\left(\sqrt{Rg}\right)^2 + Mg\left(2R\right) \ , \tag{7}$$

$$Mgh = \left[\frac{1}{2} + 2\right]MgR \ . \tag{8}$$

Therefore,

$$h = \frac{5}{2}R \ . \tag{9}$$

b) As we now know, the radial acceleration is given by

$$h = \frac{5}{2}\left(20 \ m\right) = 50 \ m \equiv 164 \ ft \ . \tag{10}$$

17.) Solution:

a) Since the normal force is always perpendicular to the path of motion, it does no work. Also, since the static frictional force does not displace in physical thing, it does no work. Again, we have a situation where we can use the conservation of mechanical energy as the only force doing work is the conservative gravitational force. We define the gravitational potential energy to be zero at the later position of the center of mass of the sphere. And since the system begins from rest, we can write

$$0 + Mg\ell \sin\theta = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + 0 \ . \tag{1}$$

For a uniform sphere,

$$I = \left(2/5\right)MR^2 \ . \tag{2}$$

And, if there is no slipping, then

$$v = R\omega \,. \tag{3}$$

Using equations (2) and (3), we can rewrite equation (1) as

$$Mg\ell \sin\theta = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}\left[1+\frac{2}{5}\right]Mv^2 = \frac{1}{2}\left(\frac{7}{5}\right)Mv^2 \,. \tag{4}$$

So,

$$v = \sqrt{2\left[(5/7)g \sin\theta\right]\ell}$$

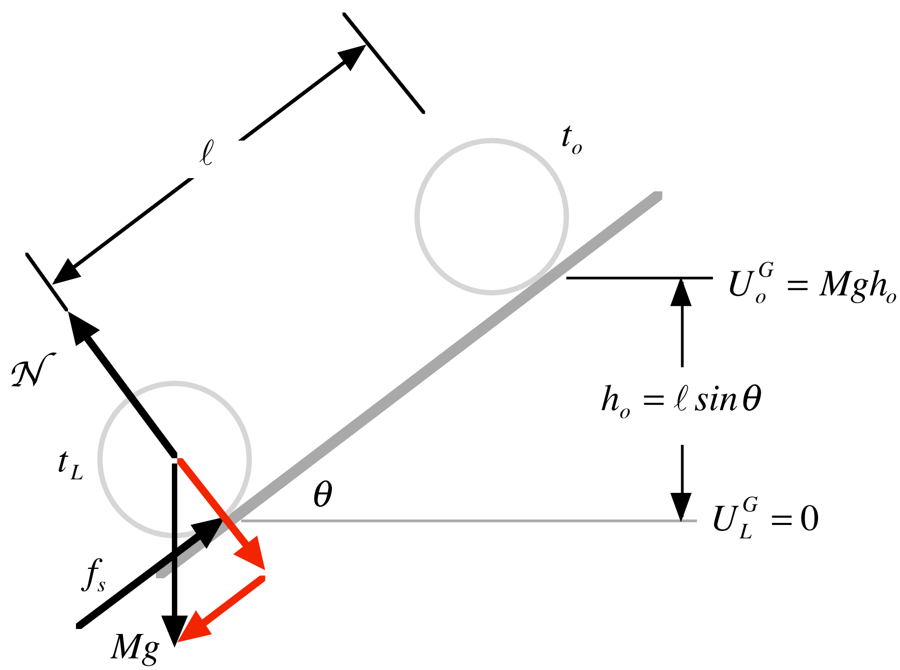
$$= \sqrt{2\left[(5/7)\left(9.805\ m/s^2\right)\left(\sin 36.87°\right)\right](1.36\ m)} = 3.381\ m/s \,. \tag{5}$$

Note: The term in the bracket in equation (6) is the magnitude of the acceleration of the center of mass of the sphere relative to the incline:

$$a = (5/7)\,g \sin\theta \,. \tag{6}$$

Recall that $g \sin\theta$ is the acceleration of an object on an inclined plane if there is no friction.

External Forces



b) We have found that a solid sphere, if released from rest on a plane inclined to the horizontal at an incline θ, will, after rolling without slipping a distance ℓ, have an instantaneous speed v given by equation (5). We know want to find out how far the sphere will have rolled when it has reached a specific fractional part of the later speed v. Using equation (5), we can write

$$v^2 = 2\left[\frac{5}{7}g \sin\theta\right]\ell \,, \tag{7}$$

and, therefore,

$$\ell = \frac{7}{10}\frac{v^2}{g\,sin\theta} \quad . \tag{8}$$

i) For a speed $v_1 = (1/4)v$, we have

$$\ell_1 = \frac{7}{10}\frac{((1/4)v)^2}{g\,sin\theta} = \frac{1}{16}\left[\frac{7v^2}{10g\,sin\theta}\right] = \frac{1}{16}\ell = \frac{1}{16}(1.360\ m) = 0.085\ m \quad . \tag{9}$$

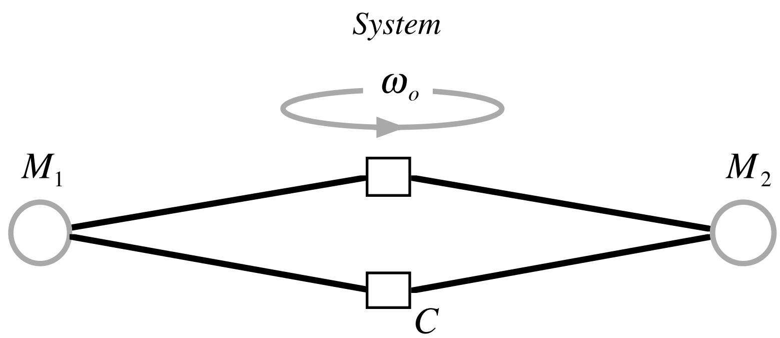
ii) For a speed $v_2 = (1/2)v$, we have

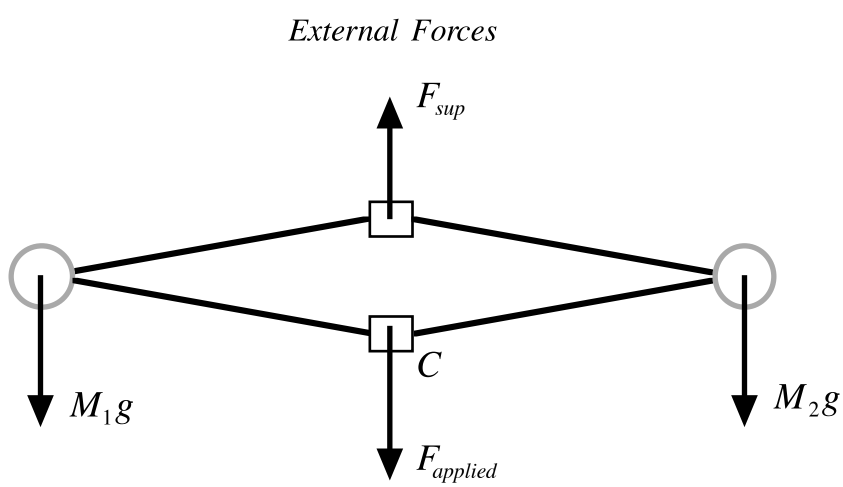
$$\ell_2 = \frac{7}{10}\frac{((1/2)v)^2}{g\,sin\theta} = \frac{1}{4}\left[\frac{7v^2}{10g\,sin\theta}\right] = \frac{1}{4}\ell = \frac{1}{4}(1.360\ m) = 0.340\ m \quad . \tag{10}$$

iii) For a speed $v_3 = (3/4)v$, we have

$$\ell_3 = \frac{7}{10}\frac{((3/4)v)^2}{g\,sin\theta} = \frac{9}{16}\left[\frac{7v^2}{10g\,sin\theta}\right] = \frac{9}{16}\ell = \frac{9}{16}(1.360\ m) = 0.765\ m \quad . \tag{11}$$

18.) Solution:



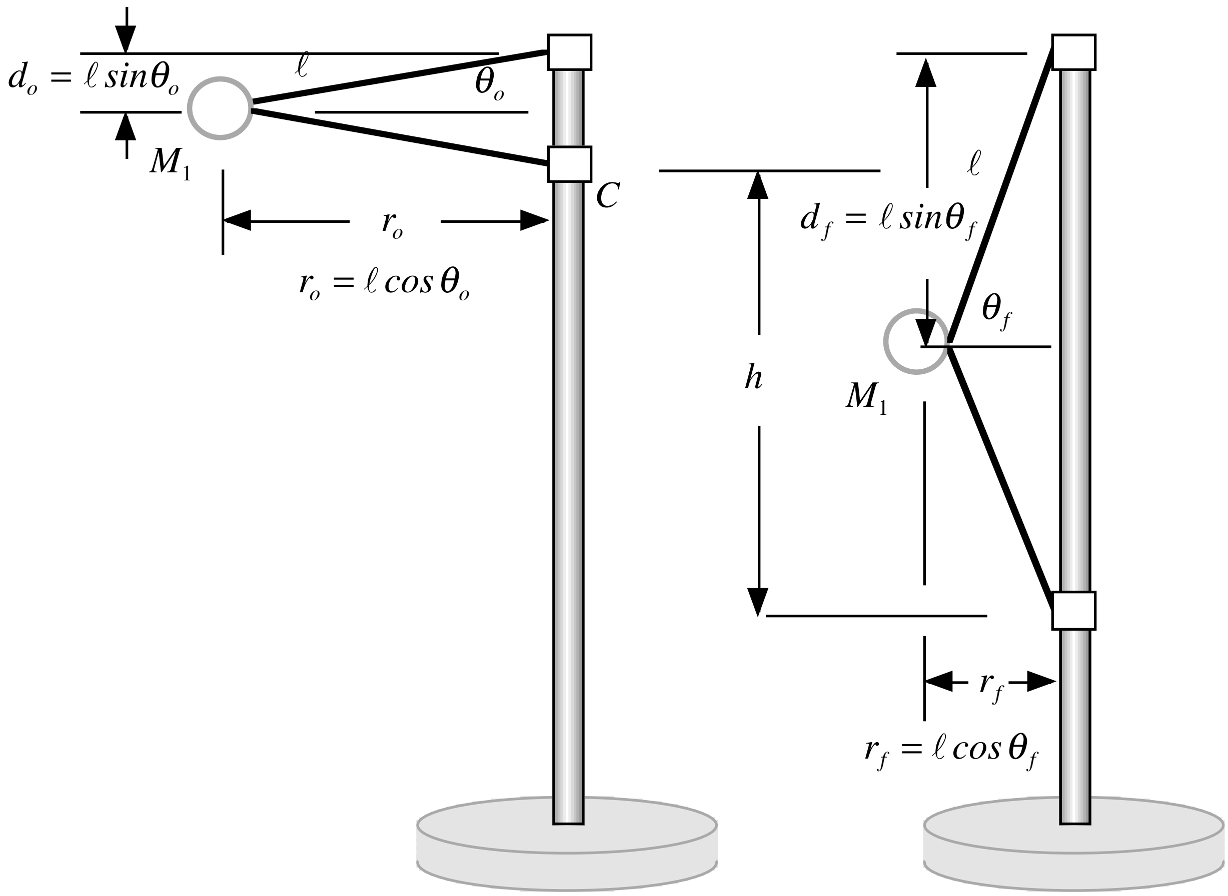


Since the support force does not displace any physical thing, it does no work. So, we can write

$$W_{F_{applied}} + W_G = \Delta K \ , \tag{1}$$

and

$$W_{F_{applied}} = \Delta K - W_G = K_f - K_o - \left(U_o^G - U_f^G\right) = \left(K_f + U_f^G\right) - \left(K_o + U_o^G\right) \ . \tag{2}$$



Note: Initially, the vertical distance from the top to the level of the balls is

$$d_o = \ell\, sin\theta_o = \ell\left[\frac{\sqrt{\ell^2 - r_o^2}}{\ell}\right] = \sqrt{\ell^2 - r_o^2} \ . \tag{3}$$

Later, this distance is given by

$$d_f = \ell\, sin\theta_f = \ell\left[\frac{\sqrt{\ell^2 - r_f^2}}{\ell}\right] = \sqrt{\ell^2 - r_f^2} \ . \tag{4}$$

However,

$$2d_o + h = 2d_f \ , \tag{5}$$

and, therefore,

$$h = 2\left(d_f - d_o\right) = 2\left[\sqrt{\ell^2 - r_f^2} - \sqrt{\ell^2 - r_o^2}\right]$$

$$= 2\left[\sqrt{(0.1523\ m)^2 - (0.05\ m)^2} - \sqrt{(0.1523\ m)^2 - (0.15\ m)^2}\right]$$

$$= 2[0.1439\ m - 0.02637\ m] = 0.2351\ m\ . \tag{6}$$

Also, note that during the application of the applied force, there is no net external torque exerted on the system. Therefore, the angular momentum is conserved and we can write

$$I_o \omega_o = I_f \omega_f\ , \tag{7}$$

and

$$2\left(r_o^2 M_1\right)\omega_o = 2\left(r_f^2 M_1\right)\omega_f\ , \tag{8}$$

so

$$\omega_f = \left[\frac{r_o}{r_f}\right]^2 \omega_o = (3)^2 (4\ rad/s) = 36\ rad/s\ . \tag{9}$$

We now return to equation (2), and we arbitrarily define the later gravitational potential energy to be zero. Equation (2) becomes

$$W_{F_{applied}} = \left(K_f + 0\right) - \left(K_o + U_o^G\right)$$

$$= 2\left[\frac{1}{2}M_1 v_f^2\right] - 2\left[\frac{1}{2}M_1 v_o^2\right] - 2M_1 g\left[\frac{h}{2}\right] + 0$$

$$= 2\left[\frac{1}{2}M_1 v_f^2\right] - 2\left[\frac{1}{2}M_1 v_o^2\right] - 2M_1 g\frac{h}{2} + 0 = M_1\left[v_f^2 - v_o^2 - gh\right]\ . \tag{10}$$

Finally, we can solve for the work done by the applied force. We have

$$W_{F_{applied}} = M_1\left[\left(r_f\omega_f\right)^2 - \left(r_o\omega_o\right)^2 - gh\right]$$

$$= (0.50\ kg)\Big[\left((.05\ m)(36\ rad/s)\right)^2 - \left((.15\ m)(4\ rad/s)\right)^2$$

$$-\left(9.805\ m/s^2\right)(0.235\ m)\Big]$$

$$= 0.287\ J\ . \tag{11}$$