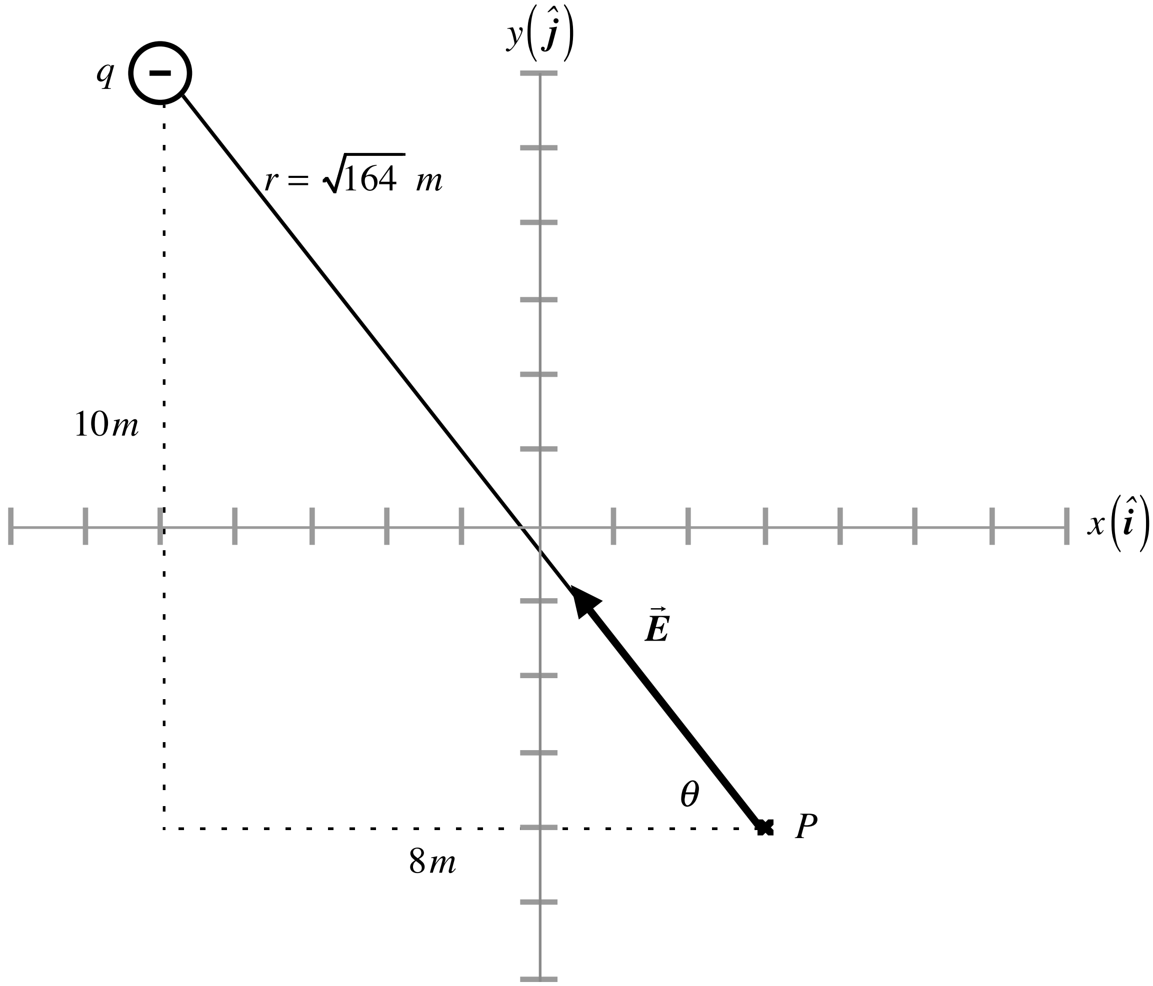
Answers to PHY2054 Practice Exam I Spring 2012

1.)



a) The magnitude of the electric field produced by an electric point charge is given by

$$E = \left|\frac{kq}{r^2}\right| = \left|\frac{\left(8.99\times 10^9\ Nm^2/C^2\right)\left(-6.75\times 10^{-9}C\right)}{\left(\sqrt{164}\ m\right)^2}\right| = 0.370\ \frac{N}{C}\ . \tag{1}$$

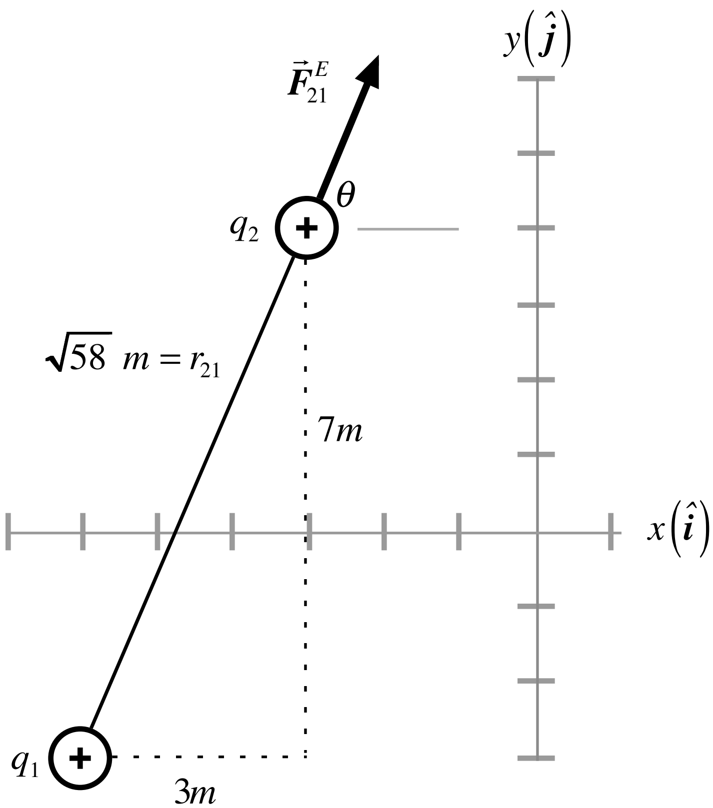
b) The unit vector that represents the direction of the electric field produced by charge q at point P is given by

$$\hat{E} = -\cos\theta\ \hat{i} + \sin\theta\ \hat{j} = -\frac{8}{\sqrt{164}}\ \hat{i} + \frac{10}{\sqrt{164}}\ \hat{j} = -0.6247\ \hat{i} + 0.7809\ \hat{j}\ . \tag{2}$$

c) The angle θ is given by

$$\theta = tan^{-1}\left|\frac{10}{8}\right| = 51.3°\,. \tag{3}$$

2.)



a) The magnitude of the electric force exerted on electric charge two by electric charge one is given by

$$F_{21}^{E} = \left|\frac{kq_2q_1}{r_{21}^2}\right| = \left|\frac{\left(8.99\times10^{9}\,Nm^2/C^2\right)\left(3.21\times10^{-6}C\right)\left(9.63\times10^{-6}C\right)}{\left(\sqrt{58}\ m\right)^2}\right|$$

$$= 4.79\times10^{-3}\,N\,. \tag{1}$$

b) The unit vector that represents the direction of the force vector is given by

$$\hat{F}_{21}^{E} = cos\,\theta\ \hat{i} + sin\theta\ \hat{j} = \frac{3}{\sqrt{58}}\ \hat{i} + \frac{7}{\sqrt{58}}\ \hat{j} = 0.3939\ \hat{i} + 0.9191\ \hat{j}\ . \tag{2}$$

c) Angle θ is given by

$$\theta = cos^{-1}\left[0.3939\right] = 66.8° = tan^{-1}\left[7/3\right] = sin^{-1}\left[0.9191\right]. \tag{3}$$

3.) The electric field at point P due to the dipole is given by

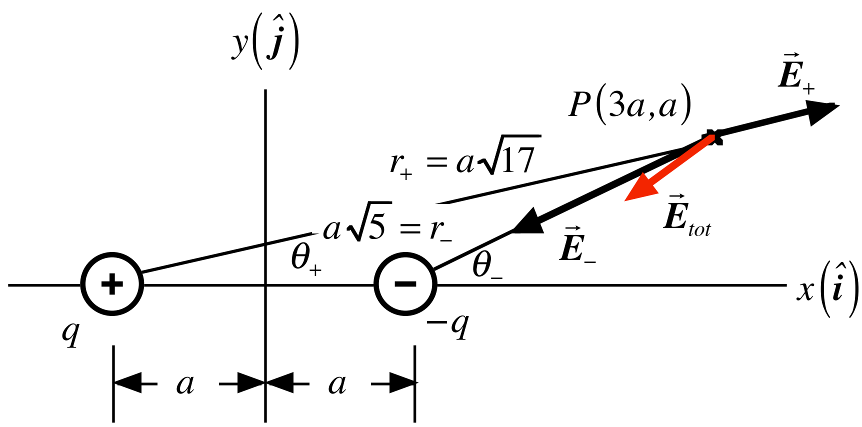
$$\vec{E}_{tot} = \vec{E}_{+} + \vec{E}_{-}$$

$$= \left|\frac{kq}{r_+^2}\right|\left[cos\,\theta_+\ \hat{i} + sin\,\theta_+\ \hat{j}\right] + \left|\frac{kq}{r_-^2}\right|\left[-cos\,\theta_-\ \hat{i} - sin\,\theta_-\ \hat{j}\right]$$

$$= \left|\frac{kq}{17a^2}\right|\left[\frac{4}{\sqrt{17}}\ \hat{i} + \frac{1}{\sqrt{17}}\ \hat{j}\right] + \left|\frac{kq}{5a^2}\right|\left[-\frac{2}{\sqrt{5}}\ \hat{i} - \frac{1}{\sqrt{5}}\ \hat{j}\right]$$

$$= \left|\frac{kq}{a^2}\right|\left[\left(\frac{4}{17\sqrt{17}} - \frac{2}{5\sqrt{5}}\right)\hat{\boldsymbol{i}} + \left(\frac{1}{17\sqrt{17}} - \frac{1}{5\sqrt{5}}\right)\hat{\boldsymbol{j}}\right]$$

$$= \left|\frac{kq}{a^2}\right|\left[-0.12182\ \hat{\boldsymbol{i}} - 0.07518\ \hat{\boldsymbol{j}}\right]. \quad (1)$$



So, the magnitude of the net electric field at point P is given by

$$E_{tot} = \left|\frac{kq}{a^2}\right|\sqrt{(-0.12182)^2 + (-0.07518)^2} = 0.1432\left|\frac{kq}{a^2}\right|$$

$$= 0.1432\left|\frac{(8.99\times10^9\ Nm^2/C^2)(2.50\times10^{-9}\ C)}{(5.25\times10^{-3}\ m)^2}\right| = 1.168\times10^5\ \frac{N}{C}\ . \quad (2)$$

b) The unit vector that represents the direction of the total electric field is given by

$$\hat{\boldsymbol{E}}_{tot} = \frac{\vec{\boldsymbol{E}}_{tot}}{E_{tot}} = -\frac{0.12182}{0.1432}\ \hat{\boldsymbol{i}} - \frac{0.07518}{0.1432}\ \hat{\boldsymbol{j}} = -0.8507\ \hat{\boldsymbol{i}} - 0.5250\ \hat{\boldsymbol{j}}\ . \quad (3)$$

c) We can now write

$$\theta_x = cos^{-1}[-0.8507] = 148.3°\ , \quad (4)$$

while

$$\theta_y = cos^{-1}[-0.5250] = 121.7°\ . \quad (5)$$

4.) a) The net electrical force exerted on the "top" electron is given by

$$\vec{\boldsymbol{F}}^E_{e_t^-,tot} = \vec{\boldsymbol{F}}^E_{e_t^-,nuc} + \vec{\boldsymbol{F}}^E_{e_t^-,e_b^-} = -\frac{2ke^2}{r^2}\ \hat{\boldsymbol{j}} + \frac{ke^2}{(2r)^2}\ \hat{\boldsymbol{j}} = -\frac{ke^2}{r^2}\left[2 - \frac{1}{4}\right]\hat{\boldsymbol{j}}$$

$$= -\frac{7}{4}\frac{ke^2}{r^2}\ \hat{\boldsymbol{j}} = -\frac{7}{4}\frac{(8.99\times10^9\ Nm^2/C^2)(1.602\times10^{-19}\ C)^2}{(7.50\times10^{-11}\ m)^2}\ \hat{\boldsymbol{j}}$$

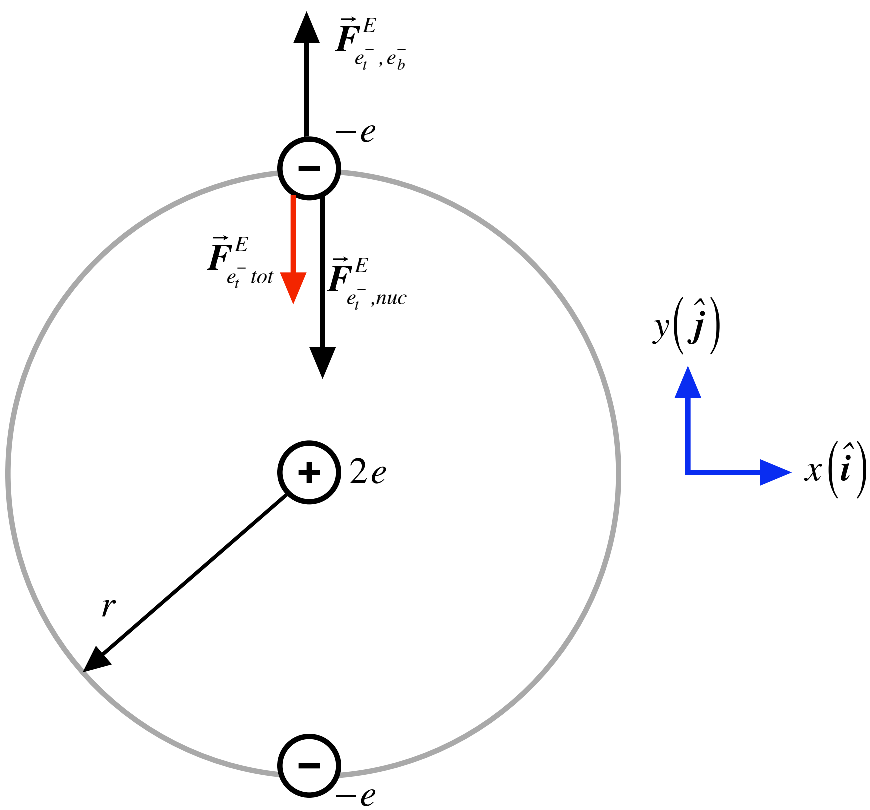
$$= -7.18\times10^{-8}\ N\ \hat{\boldsymbol{j}}\ . \quad (1)$$

b) For a circular orbit, we can write

$$\frac{7}{4}\frac{ke^2}{r^2}=\frac{M_{e^-}v^2}{r} , \quad (2)$$

and, therefore,

$$v=\sqrt{\frac{7}{4}\frac{ke^2}{M_{e^-}r}}=\sqrt{\frac{7}{4}\frac{\left(8.99\times10^9 Nm^2/C^2\right)\left(1.602\times10^{-19}C\right)^2}{\left(9.1\times10^{-31}kg\right)\left(7.50\times10^{-11}m\right)}}=2.43\times10^6\ \frac{m}{s} . \quad (3)$$



5.) a) The magnitude of the electric field produced by this point charge at point P is given by

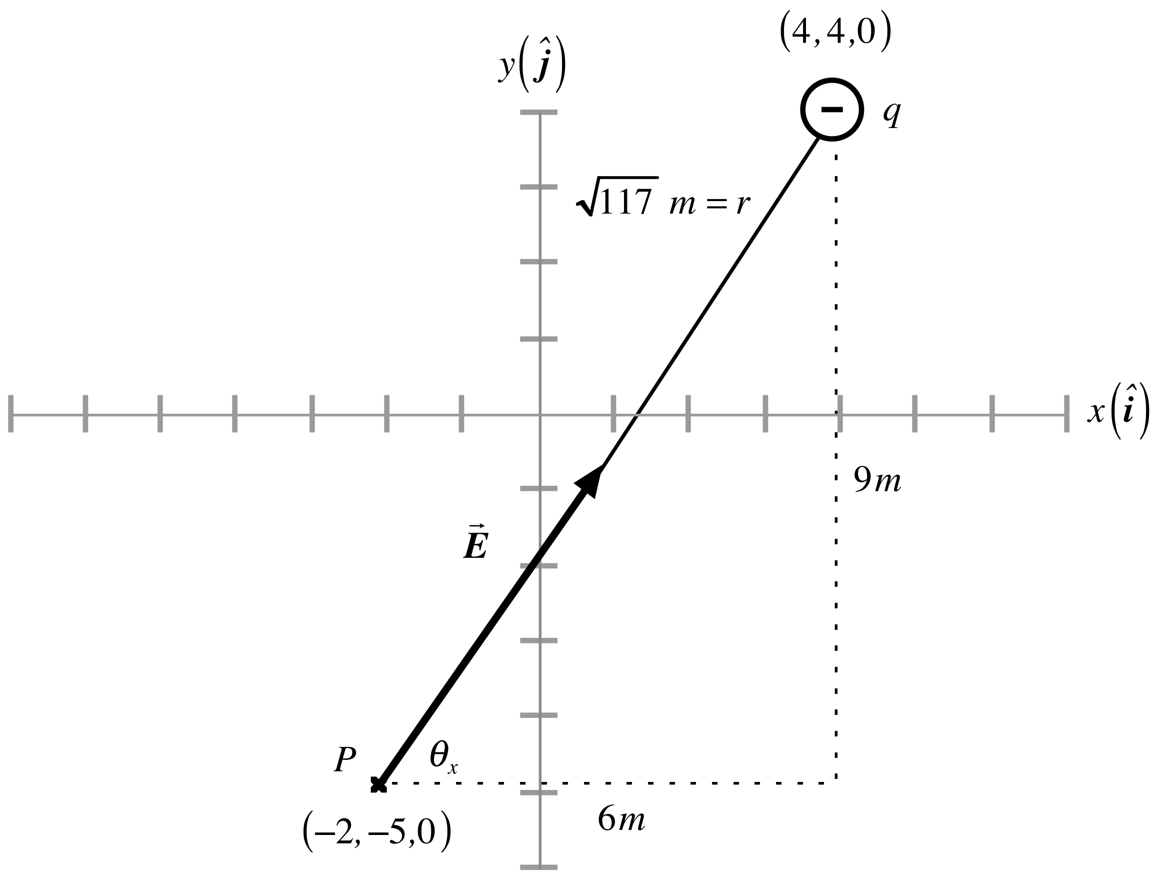
$$E=\left|\frac{kq}{r^2}\right|=\left|\frac{\left(8.99\times10^9 Nm^2/C^2\right)\left(-7.75\times10^{-9}C\right)}{\left(\sqrt{117}\ m\right)^2}\right|=0.595\ \frac{N}{C} . \quad (1)$$

b) The unit vector that represents the direction of the electric field at point P is given by

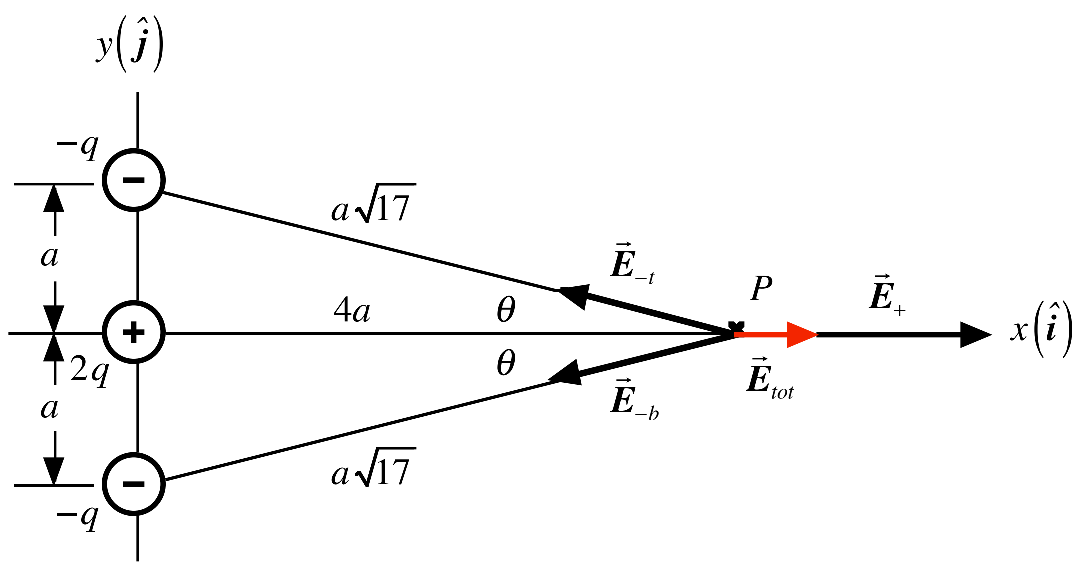
$$\hat{E}=cos\,\theta_x\ \hat{i}+sin\theta_x\ \hat{j}=\frac{6}{\sqrt{117}}\ \hat{i}+\frac{9}{\sqrt{117}}\ \hat{j}=0.5547\ \hat{i}+0.83205\ \hat{j} . \quad (2)$$

c) The angle θ_x is given by

$$\theta_x=cos^{-1}\left[0.5547\right]=56.3° . \quad (3)$$



6.)



a) The electric field produced at point P by the quadrupole is given by

$$\vec{E}_{tot} = \vec{E}_+ + \vec{E}_{-t} + \vec{E}_{-b} = \vec{E}_+ + 2\vec{E}_{-x,t} = E_+ \ \hat{i} - 2\left[E_{-t} \cos\theta\right] \hat{i}$$

$$= \frac{2kq}{(4a)^2} \ \hat{i} - 2\left[\frac{kq}{17a^2}\frac{4}{\sqrt{17}}\right] \hat{i} = \frac{2kq}{a^2}\left[\frac{1}{16} - \frac{4}{17^{3/2}}\right] \hat{i} = (0.010866)\frac{kq}{a^2} \ \hat{i} \ . \qquad (1)$$

The magnitude, then, is given by

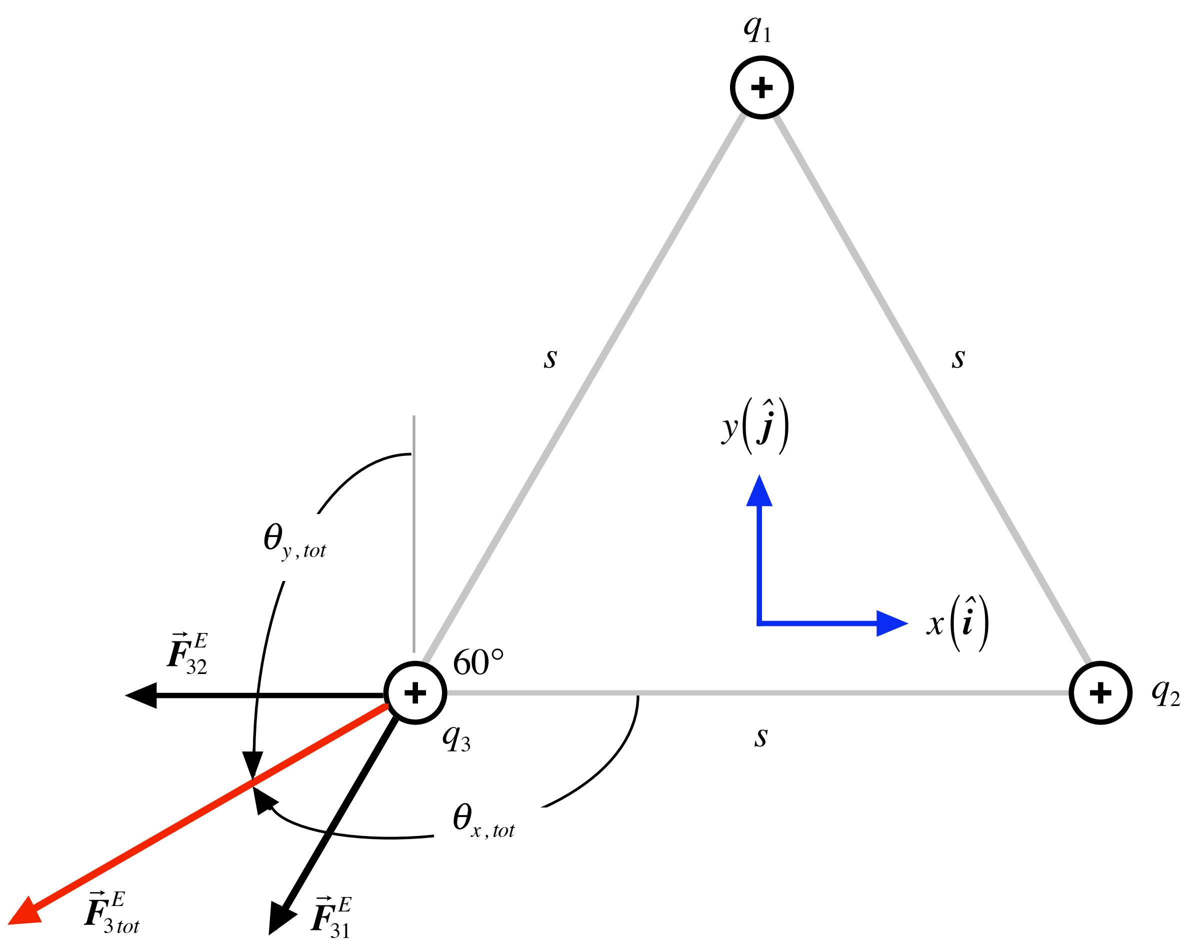
$$E_{tot} = (0.010866)\frac{kq}{a^2} = (0.010866)\frac{(8.99\times10^{-9}\ Nm^2/C^2)(7.50\times10^{-9}\ C)}{(2.25\times10^{-3}\ m)^2}$$

$$= 1.447\times10^5\ \ N/C\ . \qquad (2)$$

b) Of course, the direction of the net electric field at point P is given by

$$\hat{\boldsymbol{E}}_{tot} = \hat{\boldsymbol{i}}\ . \qquad (3)$$

7.)



a) The net electric force exerted on charge three is given by

$$\vec{\boldsymbol{F}}^E_{3tot} = \vec{\boldsymbol{F}}^E_{31} + \vec{\boldsymbol{F}}^E_{32} = \left|\frac{kq^2}{s^2}\right|\left[-cos\,60°\ \hat{\boldsymbol{i}} - sin\,60°\ \hat{\boldsymbol{j}}\right] - \left|\frac{kq^2}{s^2}\right|\hat{\boldsymbol{i}}$$

$$= \left|\frac{kq^2}{s^2}\right|\left[-(1+cos\,60°)\ \hat{\boldsymbol{i}} - sin\,60°\ \hat{\boldsymbol{j}}\right]\ . \qquad (1)$$

Therefore, the magnitude of the net electric force on charge three is given by

$$F^E_{3tot} = \left|\frac{kq^2}{s^2}\right|\sqrt{\left(-(1+cos\,60°)\right)^2 + (sin\,60°)^2} = \sqrt{3}\left|\frac{kq^2}{s^2}\right|$$

$$= \frac{\sqrt{3}\left(8.99\times10^{9}\ Nm^2/C^2\right)\left(6.275\times10^{-6}\ C\right)^2}{\left(0.225\ m\right)^2} = 12.11\ N\ . \qquad (2)$$

b) The unit vector that represents the direction of the net electric force on charge three is given by

$$\hat{\boldsymbol{F}}^E_{3tot} = -\frac{\left(1+cos\,60°\right)}{\sqrt{3}}\ \hat{\boldsymbol{i}} - \frac{sin\,60°}{\sqrt{3}}\ \hat{\boldsymbol{j}} = -0.8660\ \hat{\boldsymbol{i}} - 0.5000\ \hat{\boldsymbol{j}}\ . \qquad (3)$$

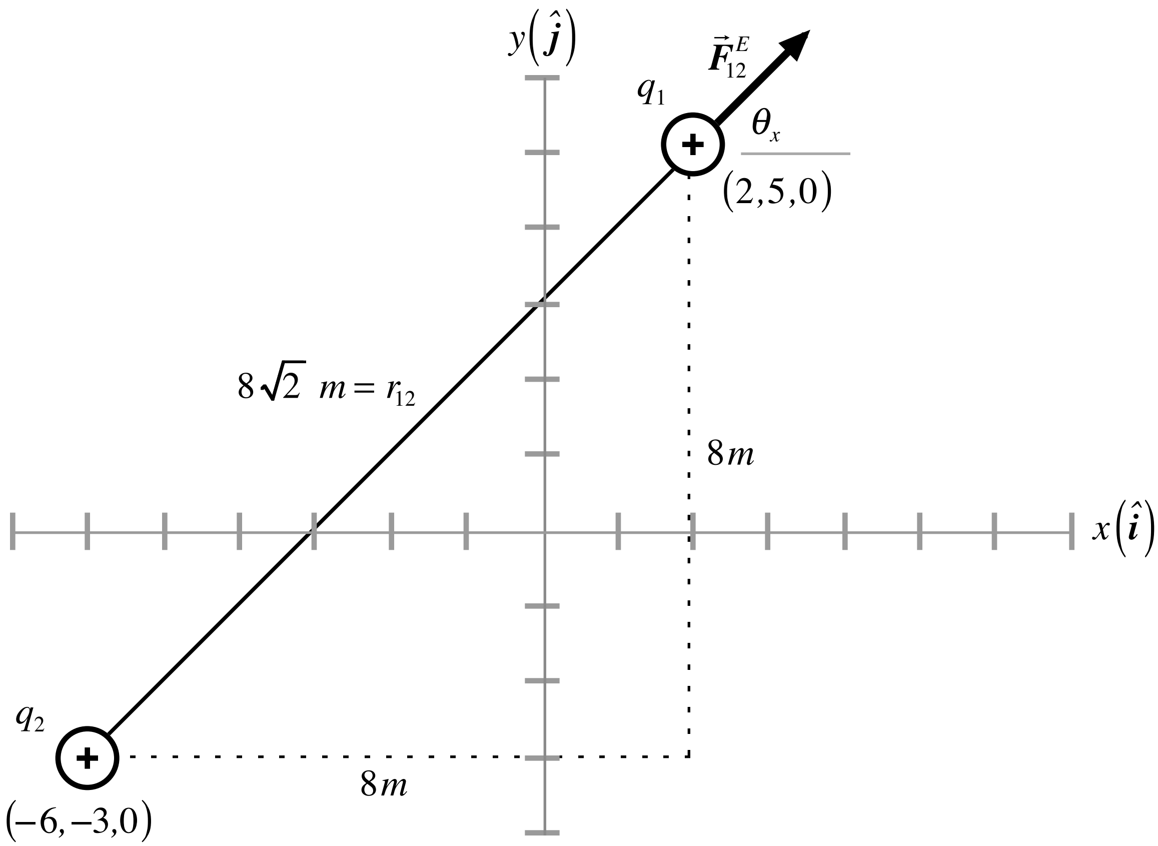
c) So, $\theta_{x,tot}$ is given by

$$\theta_{x,tot} = cos^{-1}\left[-0.8660\right] = 150°\ , \qquad (4)$$

while $\theta_{y,tot}$ is given by

$$\theta_{y,tot} = cos^{-1}\left[-0.5000\right] = 120°\ . \qquad (5)$$

8.)



a) The magnitude of the electric force exerted on charge one is given by

$$F^E_{12} = \left|\frac{kq_1q_2}{r^2_{12}}\right| = \left|\frac{\left(8.99\times10^{9}\ Nm^2/C^2\right)\left(8.36\times10^{-6}C\right)\left(4.18\times10^{-6}\ C\right)}{\left(8\sqrt{2}\ m\right)^2}\right|$$

$$= 2.454\times10^{-3}\ N\ . \qquad (1)$$

b) The unit vector that represents the direction of the electric force exerted on charge one is given by

$$\hat{\boldsymbol{F}}_{12}^{E} = cos\,\theta_x \;\hat{\boldsymbol{i}} + sin\,\theta_x \;\hat{\boldsymbol{j}} = \frac{8}{8\sqrt{2}}\,\hat{\boldsymbol{i}} + \frac{8}{8\sqrt{2}}\,\hat{\boldsymbol{j}} = 0.7071\;\hat{\boldsymbol{i}} + 0.7071\;\hat{\boldsymbol{j}}\ . \qquad (2)$$

c) Therefore, the measure of angle θ_x is given by

$$\theta_x = cos^{-1}\left[0.7071\right] = 45^\circ\ . \qquad (3)$$