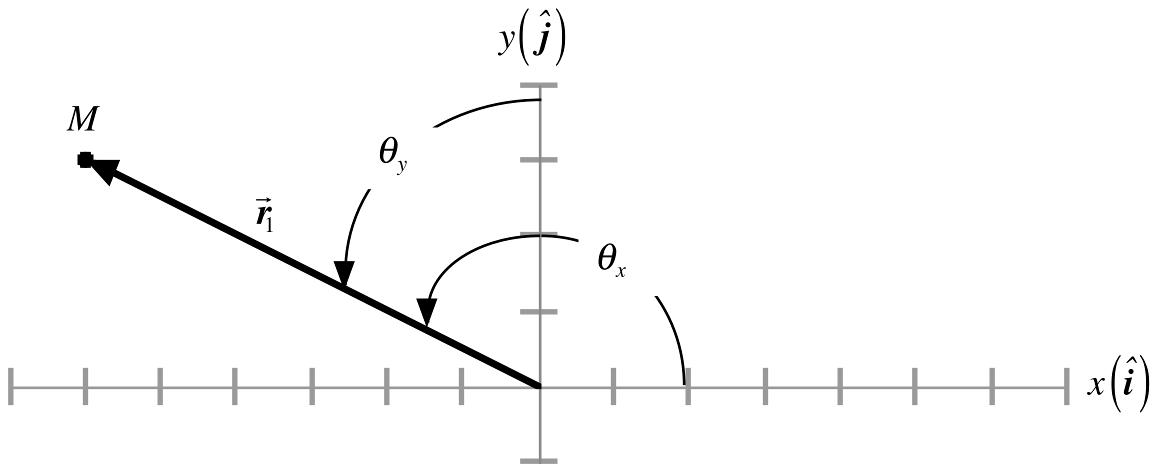
Answers to PHY2053 Practice Exam I Spring 2012

1.)



a) The magnitude of this position vector is given by

$$r_1 = \sqrt{(-6.00\ m)^2 + (3.00\ m)^2} = \sqrt{45}\ m = 6.708\ m\ . \qquad (1)$$

b) The unit vector that represents the direction of this position vector is given by

$$\hat{r}_1 = \frac{\vec{r}_1}{r_1} = \frac{-6.00}{\sqrt{45}}\ \hat{i} + \frac{3.00}{\sqrt{45}}\ \hat{j} = -0.8944\ \hat{i} + 0.4472\ \hat{j}$$

$$= cos\theta_x\ \hat{i} = cos\theta_y\ \hat{j}\ . \qquad (2)$$

c) The angle this position vector makes to the positive branch of the x-axis is θ_x and it is given by

$$\theta_x = cos^{-1}\left[-\frac{6}{\sqrt{45}}\right] = 153.4°\,. \qquad (3)$$

d) The angle this position vector makes to the positive branch of the y-axis is θ_y and it is given by

$$\theta_y = cos^{-1}\left[\frac{3}{\sqrt{45}}\right] = 63.4°\,. \qquad (4)$$

2.) b) The change in position vector is given by

$$\Delta\vec{r} = \vec{r}_L - \vec{r}_o = \left[-2.00\ m\ \hat{i} + 6.00\ m\ \hat{j}\right] - \left[-5.00\ m\ \hat{i} - 4.00\ m\ \hat{j}\right]$$

$$= 3.00\ m\ \hat{i} + 10\ m\ \hat{j}\ . \qquad (1)$$

So, the magnitude of this change in position vector is given by

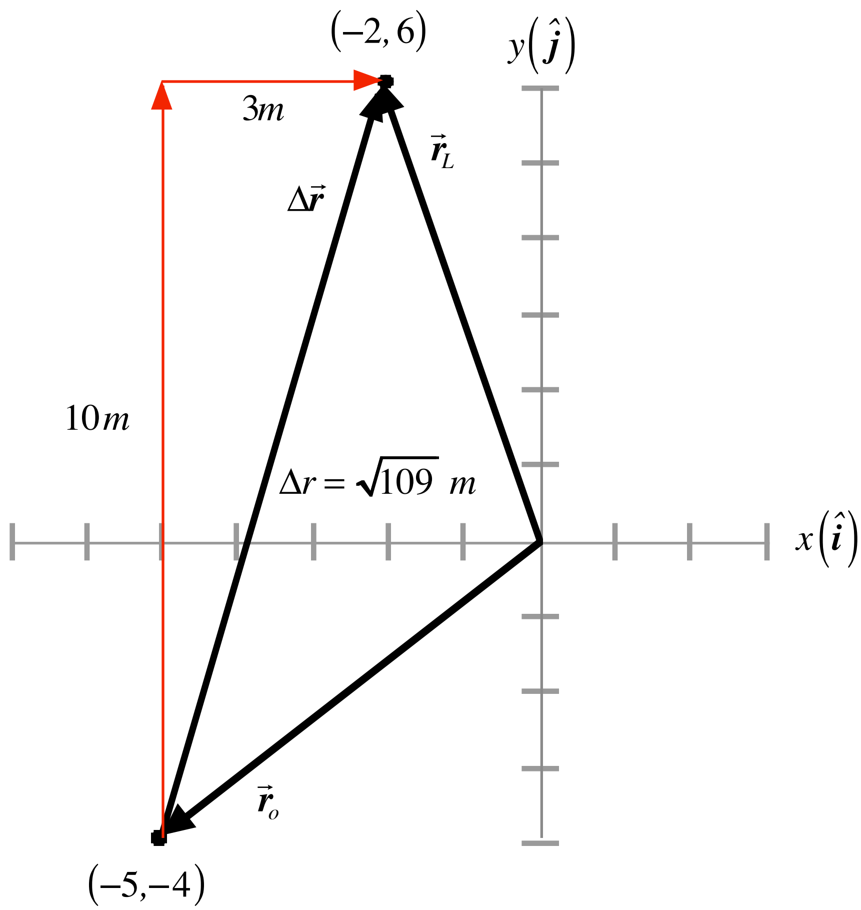
$$\Delta r = \sqrt{(3.00\ m)^2 + (10\ m)^2} = \sqrt{109}\ m = 10.44\ m\,. \qquad (2)$$

c) The unit vector that represents the direction of this change in position vector is given by

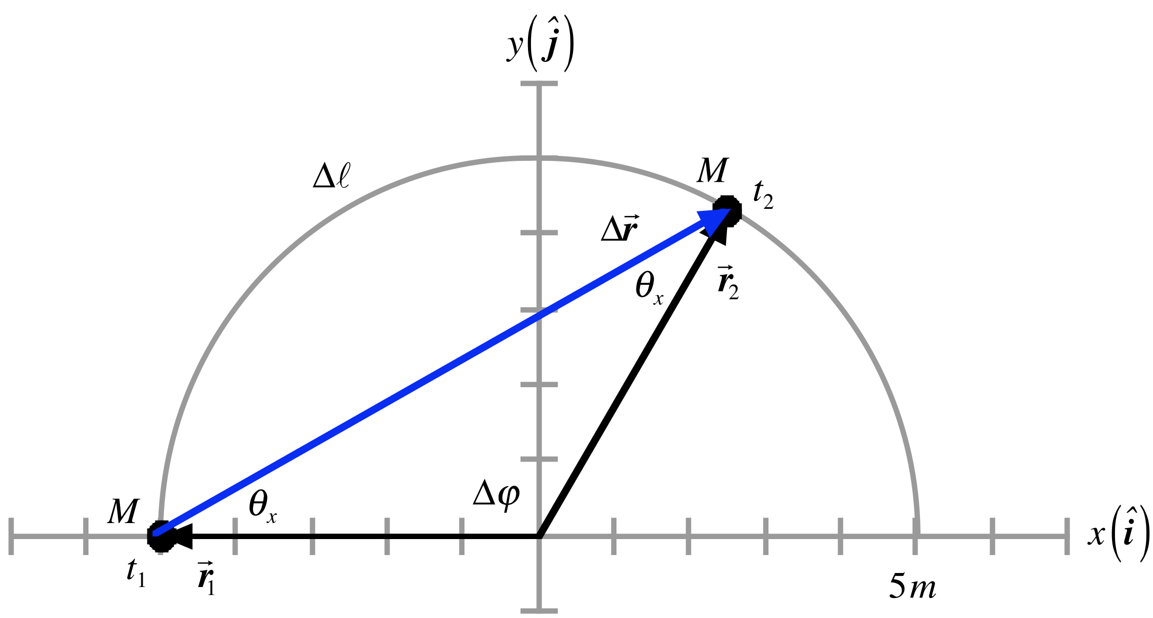
$$\hat{\Delta r} = \frac{\Delta\vec{r}}{\Delta r} = \frac{3}{\sqrt{109}}\ \hat{i} + \frac{10}{\sqrt{109}}\ \hat{j} = 0.2873\ \hat{i} + 0.9578\ \hat{j}$$

$$= cos\theta_x \ \hat{\boldsymbol{i}} = cos\theta_y \ \hat{\boldsymbol{j}} \ . \tag{3}$$

a)



3.)



a) To find the average velocity, we first need the change in position. To that end, we write

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \left[2.5000\ m\ \hat{i} + 4.3301\ m\ \hat{j}\right] - \left[-5.00\ m\ \hat{i}\right]$$

$$= 7.5000\ \hat{i} + 4.3301\ m\ \hat{j}\ . \qquad (1)$$

So, the average velocity is given by

$$\vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t} = \frac{= 7.5000\ \hat{i} + 4.3301\ m\ \hat{j}}{2\ s} = 3.750\ \frac{m}{s}\ \hat{i} + 2.16505\ \frac{m}{s}\ \hat{j}\ . \qquad (2)$$

b) The magnitude of the average velocity is given by

$$v_{ave} = \sqrt{(3.750)^2 + (2.16505\ m/s)^2} = 4.330\ m/s\ . \qquad (3)$$

c) The unit vector that represents the direction of the average velocity is given by

$$\hat{v}_{ave} = \frac{\vec{v}_{ave}}{v_{ave}} = \frac{3.750}{4.330}\hat{i} + \frac{2.1605}{4.330}\ \hat{j} = 0.8661\ \hat{i} + 0.4990\ \hat{j}$$

$$= cos\theta_x\ \hat{i} = cos\theta_y\ \hat{j}\ . \qquad (4)$$

d) For circular arc lengths, we can use

$$\Delta\ell = R(\Delta\varphi_{rad}), \qquad (5)$$

where $\Delta\varphi_{rad}$ is the central angle subtended by the arc's radii. So, we note that

$$\Delta\varphi_{rad} = 180° - 2\theta_x\ , \qquad (6)$$

where

$$\theta_x = cos^{-1}[0.8661] = 30°\ . \qquad (7)$$

Therefore,

$$\Delta\varphi_{rad} = 180° - 2(30°) = 120°. \qquad (8)$$

So, equation (5) becomes

$$\Delta\ell = R(\Delta\varphi_{rad}) = (5.00\ m)\left[120\frac{\pi}{180}\right] = 10.4720\ m\ . \qquad (9)$$

As a point of interest, the average speed is

$$s_{ave} = \frac{\Delta\ell}{\Delta t} = \frac{10.4720\ m}{2\ s} = 5.2360\ \frac{m}{s}. \qquad (10)$$

The average speed is the same as the instantaneous speed because the speed is constant.

4.) a) The average acceleration is given by

$$\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t} = \frac{1}{\Delta t}[\vec{v}_L - \vec{v}_o] = \frac{1}{(3s)}\left[\left(-8.8\ m/s\ \hat{j}\right) - \left(20.6\ m/s\ \hat{j}\right)\right]$$

$$= -9.80\ m/s^2\ \hat{j}\ . \qquad (1)$$

b) The magnitude of the average acceleration is given by

$$a_{ave} = \sqrt{\left(-9.80\ m/s^2\right)^2} = 9.80\ m/s^2\ . \qquad (2)$$

c.) The unit vector that represents the direction of the average acceleration is given by

$$\hat{a}_{ave} = \frac{\vec{a}_{ave}}{a_{ave}} = \frac{-9.80\ m/s^2\ \hat{j}}{9.80\ m/s^2} = -\ \hat{j}\ . \qquad (3)$$

d) We have

$$h = h_L - d \ , \tag{4}$$

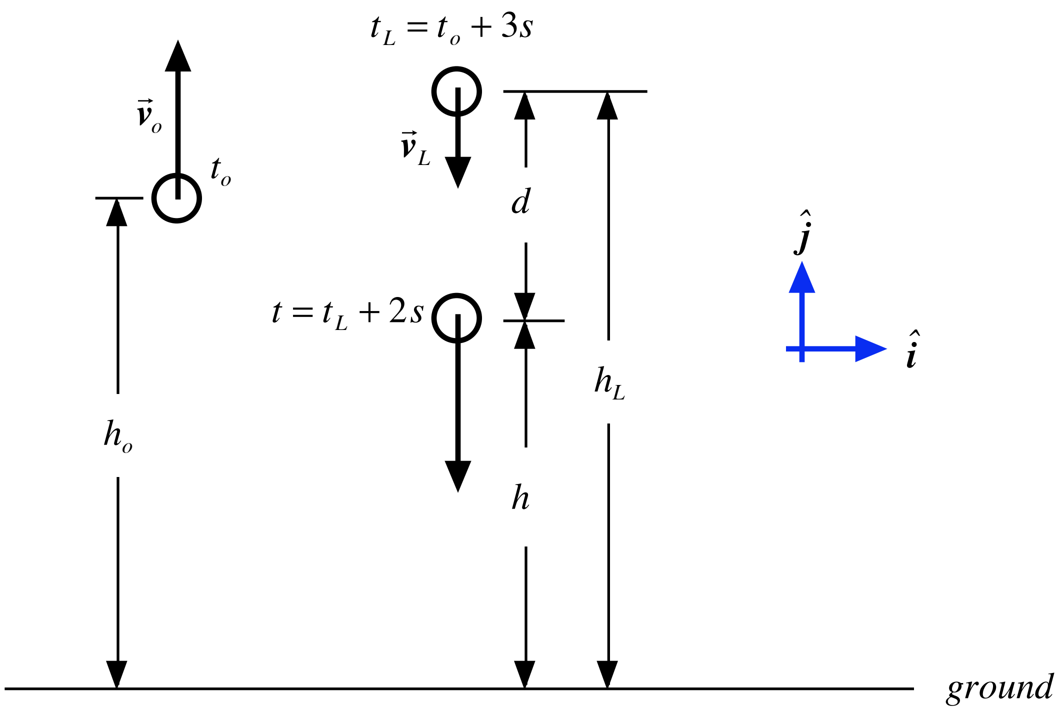
where

$$d = v_L t + \frac{1}{2} a_{ave} t^2 = (8.8\ m/s)(2\ s) + \frac{1}{2}(9.80\ m/s^2)(2\ s)^2 = 37.2\ m\ . \tag{5}$$

Therefore,

$$h = h_L - d = 123.6\ m - 37.2\ m = 86.4\ m\ . \tag{6}$$

In this solution we have assumed a constant acceleration. In such an event, the magnitude of the instantaneous acceleration is equal to the magnitude of the average acceleration. For objects close to the surface of the Earth, this is a very useful approximation.



5.) a) The magnitude of this position vector is given by

$$r_1 = \sqrt{(-3.00\ m)^2 + (-5.00\ m)^2} = \sqrt{34}\ m = 5.831\ m\ . \tag{1}$$

b) The unit vector that represents the direction of this position vector is given by

$$\hat{\boldsymbol{r}}_1 = \frac{\vec{\boldsymbol{r}}_1}{r_1} = -\frac{3.00}{\sqrt{34}}\ \hat{\boldsymbol{i}} - \frac{5.00}{\sqrt{34}}\ \hat{\boldsymbol{j}} = -0.5145\ \hat{\boldsymbol{i}} - 0.8575\ \hat{\boldsymbol{j}}$$

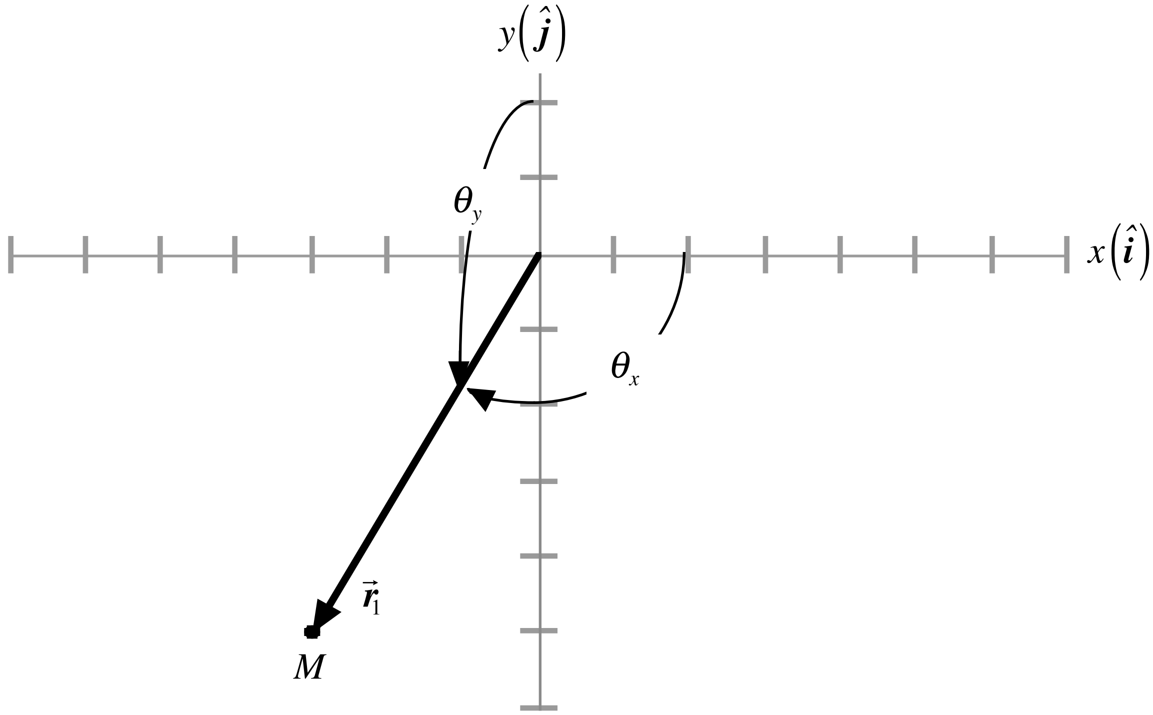
$$= cos\theta_x\ \hat{\boldsymbol{i}} = cos\theta_y\ \hat{\boldsymbol{j}}\ . \tag{2}$$

c) The angle this position vector makes to the positive branch of the x-axis is θ_x and it is given by

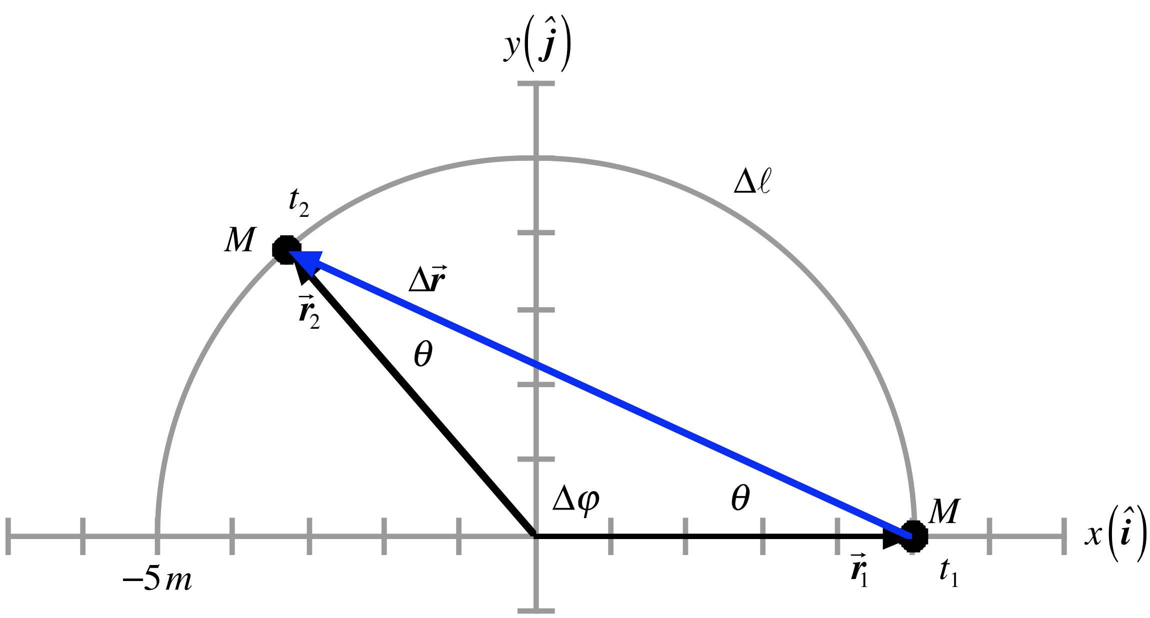
$$\theta_x = cos^{-1}\left[-\frac{3}{\sqrt{34}}\right] = 121.0°\ . \tag{3}$$

d) The angle this position vector makes to the positive branch of the y-axis is θ_y and it is given by

$$\theta_y = cos^{-1}\left[-\frac{5}{\sqrt{34}}\right] = 149.0°. \tag{4}$$



6.)



a) To find the average velocity, we first need the change in position. To that end, we write

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \left[-3.2139\ m\ \hat{i} + 3.8302\ m\ \hat{j}\right] - \left[5.00\ m\ \hat{i}\right]$$

$$=-8.2139\ \hat{\boldsymbol{i}}+3.8302\ m\ \hat{\boldsymbol{j}}\ . \qquad (1)$$

So, the average velocity is given by

$$\vec{\boldsymbol{v}}_{ave}=\frac{\Delta\vec{\boldsymbol{r}}}{\Delta t}=\frac{-8.2139\ \hat{\boldsymbol{i}}+3.8302\ m\ \hat{\boldsymbol{j}}}{2\ s}=-4.107\ \frac{m}{s}\ \hat{\boldsymbol{i}}+1.915\ \frac{m}{s}\ \hat{\boldsymbol{j}}\ . \qquad (2)$$

b) The magnitude of the average velocity is given by

$$v_{ave}=\sqrt{\left(-4.107\ m/s\right)^2+\left(1.915\ m/s\right)^2}=4.532\ m/s\ . \qquad (3)$$

c) The unit vector that represents the direction of the average velocity is given by

$$\hat{\boldsymbol{v}}_{ave}=\frac{\vec{\boldsymbol{v}}_{ave}}{v_{ave}}=-\frac{4.107}{4.532}\ \hat{\boldsymbol{i}}+\frac{1.915}{4.532}\ \hat{\boldsymbol{j}}=-0.9062\ \hat{\boldsymbol{i}}+0.42255\ \hat{\boldsymbol{j}}$$

$$=cos\theta_x\ \hat{\boldsymbol{i}}=cos\theta_y\ \hat{\boldsymbol{j}}\ . \qquad (4)$$

d) For circular arc lengths, we can use

$$\Delta\ell=R\left(\Delta\varphi_{rad}\right), \qquad (5)$$

where $\Delta\varphi_{rad}$ is the central angle subtended by the arc's radii. So, we note that

$$\Delta\varphi_{rad}=180^\circ-2\theta, \qquad (6)$$

where

$$\theta=180^\circ-\theta_{x,\ ave\ vel}$$

$$=180^\circ-cos^{-1}\left[-0.9062\right]=25^\circ\ . \qquad (7)$$

Therefore,

$$\Delta\varphi_{rad}=180^\circ-2\left(25^\circ\right)=130^\circ\ . \qquad (8)$$

So, equation (5) becomes

$$\Delta\ell=R\left(\Delta\varphi_{rad}\right)=\left(5.00\ m\right)\left[130\frac{\pi}{180}\right]=11.3446\ m\ . \qquad (9)$$

As a point of interest, the average speed is

$$s_{ave}=\frac{\Delta\ell}{\Delta t}=\frac{11.3446\ m}{2\ s}=5.6723\ \frac{m}{s}\ . \qquad (10)$$

The average speed is the same as the instantaneous speed because the speed is constant.

7.) b) The change in position vector is given by

$$\Delta\vec{\boldsymbol{r}}=\vec{\boldsymbol{r}}_L-\vec{\boldsymbol{r}}_o=\left[6.00\ m\ \hat{\boldsymbol{i}}+2.00\ m\ \hat{\boldsymbol{j}}\right]-\left[4.00\ m\ \hat{\boldsymbol{i}}-5.00\ m\ \hat{\boldsymbol{j}}\right]$$

$$=2.00\ m\ \hat{\boldsymbol{i}}+7.00\ m\ \hat{\boldsymbol{j}}\ . \qquad (1)$$

c) The magnitude of the change in position is given by

$$\Delta r=\sqrt{\left(2.00\ m\right)^2+\left(7.00\ m\right)^2}=\sqrt{53}\ m=7.280\ m\ . \qquad (2)$$

Although I did not ask for it, the unit vector that represents the direction of the change in position vector is given by

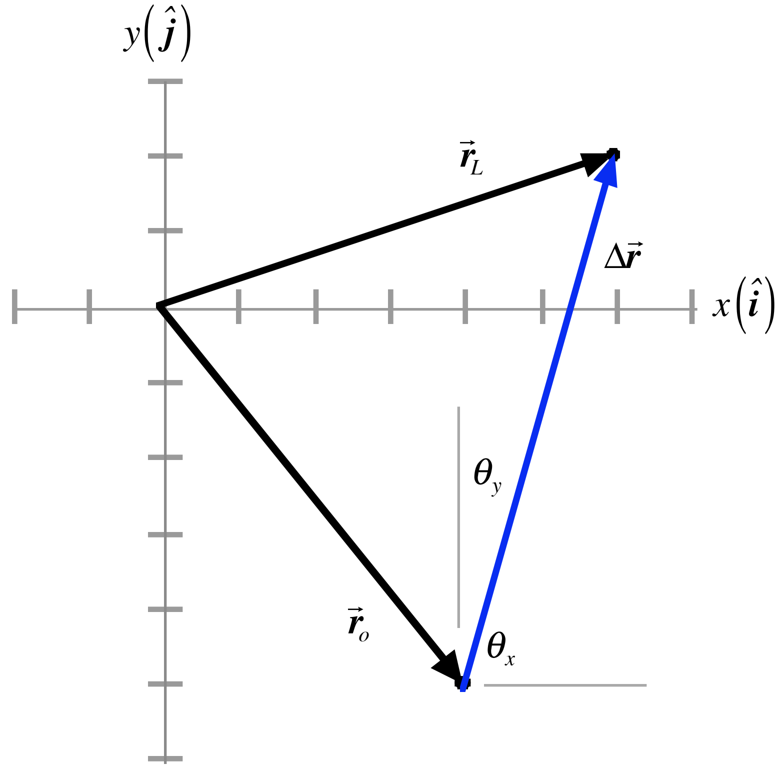
$$=\widehat{\Delta r}=\frac{\Delta\vec{\boldsymbol{r}}}{\Delta r}=\frac{2}{\sqrt{53}}\ \hat{\boldsymbol{i}}+\frac{7}{\sqrt{53}}\ \hat{\boldsymbol{j}}=0.2747\ \hat{\boldsymbol{i}}+0.9615\ \hat{\boldsymbol{j}}\ , \qquad (3)$$

which implies

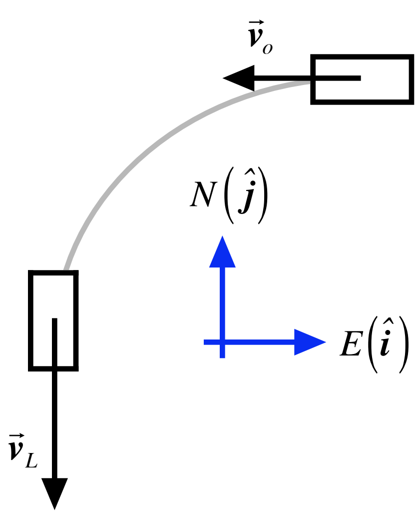
$$\theta_x = cos^{-1}\left(2/\sqrt{53}\right) = 74.1°, \tag{4}$$

and

$$\theta_y = cos^{-1}\left(7/\sqrt{53}\right) = 15.9° \ . \tag{5}$$



8.)



a) The average acceleration is given by

$$\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t} = \frac{1}{\Delta t}\left[\vec{v}_L - \vec{v}_o\right] = \frac{1}{(6s)}\left[\left(-26.822\ m/s\ \hat{j}\right) - \left(-8.941\ m/s\ \hat{i}\right)\right]$$

$$= 1.490 \ m/s^2 \ \hat{\boldsymbol{i}} - 4.470 \ m/s^2 \ \hat{\boldsymbol{j}} \ . \tag{1}$$

b) The magnitude of the average acceleration is given by

$$a_{ave} = \sqrt{\left(1.490 \ m/s^2\right)^2 + \left(-4.470 \ m/s^2\right)} = 4.712 \ m/s^2 \ . \tag{2}$$

c.) The unit vector that represents the direction of the average acceleration is given by

$$\hat{\boldsymbol{a}}_{ave} = \frac{\vec{\boldsymbol{a}}_{ave}}{a_{ave}} = \frac{1.490}{4.712} \hat{\boldsymbol{i}} - \frac{4.470}{4.712} \hat{\boldsymbol{j}} = 0.3162 \ \hat{\boldsymbol{i}} - 0.9486 \ \hat{\boldsymbol{j}} \ . \tag{3}$$